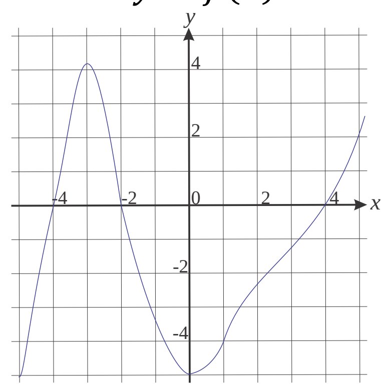
Homework for April 3, 2022.

Algebra.

Review the classwork handout. Try solving the following problems. Remember: you do not necessarily need to solve all problems, just solve as many as you can within the time you can dedicate to Math 9 homework.

1. From the picture, find in which interval(s) the function $y = f(x)$
   a. is monotonic
   b. has the same sign



2. Find all possible values of a such that equation $x^2 + ax + 9 = 0$ has two different roots, both of which are less than -1.
3. Draw graphs of the following functions
   a. $y = \left|\frac{1}{x-2} + 1\right|$
   b. $y = \frac{1}{|x|-2} + 1$
4. Solve the following equations
   a. (Skanavi 7.141) $3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^x - \frac{1}{2} \cdot 9^{x+1}$
   b. (Skanavi 7.143) $\sqrt{\log_x \sqrt{x}} = -\log_x 5$
   c. (Skanavi 7.153) $\frac{\log_2(9-2^x)}{3-x} = 1$
   d. (Skanavi 7.160) $\log_a x + \log_{a^2} x + \log_{a^3} x = 11$
   e. (Skanavi 7.184) $2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \cdots$
   f. (Skanavi 7.190) $9^x + 6^x = 2^{2x+1}$
   g. (Skanavi 7.197) $4^{\log x+1} - 6^{\log x} - 2 \cdot 3^{\log x^2+2} = 0$
   h. (Skanavi 7.299) $(x^2 - x - 1)^{x^2-1} = 1$
   i. (Skanavi 7.304) find integer root: $\log_{\sqrt{x}}(x + 12) = 8\log_{x+12} x$
   j. (Skanavi 7.308) $\log_{x+3}\left(3 - \sqrt{1 - 2x + x^2}\right) = \frac{1}{2}$
5. (Skanavi 7.277) Equation $4^x + 10^x = 25^x$ has a single root. Find this root. Is it positive or negative? Is it larger or less than 1?
6. (Skanavi 7.280) Show that:

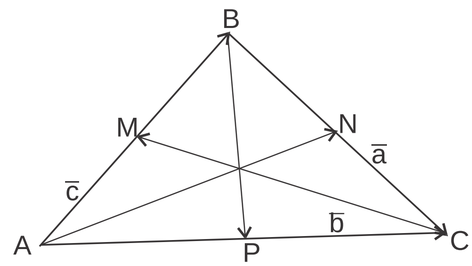
$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7 = \frac{1}{3}$$

Geometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on vectors. Solve the following problems.

Problems.

1. In a triangle ABC, vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ (**c**, **b** and **a**) are the sides. $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ are the medians.



   a. Express vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ through vectors **c**, **b** and **a.**

   b. Find the sum of vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$**.**

2. Solve the same problem for bisectors $\overrightarrow{AN}$, $\overrightarrow{CM}$ and $\overrightarrow{BP}$ in a triangle ABC.

3. Coxeter, Greitzer, problem #9 to Sec. 2.1 (p. 31): How far away is the horizon as seen from the top of a mountain 1 mile high? (Assume the Earth to be a sphere of diameter 7920 miles.)

4. In a rectangle $ABCD$, A_1, B_1, C_1 and D_1 are the mid-points of sides AB, CD, BC and DA, respectively. M is the crossing point of the segments A_1B_1, and C_1D_1, connecting two pairs of midpoints.

   a. Express vector $\overrightarrow{A_1M}$ through $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$.

   b. Prove that M is the mid-point of segments, A_1B_1 and C_1D_1, i.e. $|A_1M| = |MB_1|$ and $|C_1M| = |MD_1|$.

5. In a parallelogram $ABCD$, find $\overrightarrow{AB} + \overrightarrow{BD} - 2\overrightarrow{AD}$.

6. M is a crossing point of the medians in a triangle ABC. Prove that $\overrightarrow{AM} = \frac{1}{3}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$.

7. For three points, $A(-1,3)$, $B(2,-5)$ and $C(3,4)$, find the (coordinates of) following vectors,

   a. $\overrightarrow{AB} - \overrightarrow{BC}$

   b. $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC}$

c. $\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA}$

8. For two triangles, ABC and $A_1B_1C_1$, $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$. Prove that medians of these two triangles cross at the same point M.