Homework for May 1, 2022.

Algebra/Geometry. Complex numbers.

Please, complete the previous homework assignments. Review the classwork handout on complex numbers and complete the exercises. Solve the following problems.

Problems.

1. Find all complex numbers z such that:
   a. $z^2 = -i$
   b. $z^2 = -2 + 2i\sqrt{3}$
   c. $z^3 = i$

   Hint: write and solve equations for a, b in $z = a + bi$.

2. On the complex plane, plot all fifth order roots of 1 and all fifth order roots of -1.
3.
   a. Find all roots of the polynomial $z + z^2 + z^3 + \cdots + z^n$
   b. Without doing the long division, show that $1 + z + z^2 + \cdots + z^9$ is divisible by $1 + z + z^2 + z^3 + z^4$.

4. Find the roots of the following cubic equations by heuristic guess-and-check factorization, and using the Cardano-Tartaglia formula. Reconcile the two results.
   a. $z^3 - 7z + 6 = 0$
   b. $z^3 - 21z - 20 = 0$
   c. $z^3 - 3z = 0$
   d. $z^3 + 3z = 0$
   e. $z^3 - \frac{3}{4}z + \frac{1}{4} = 0$

5. Which transformation of the complex plane is defined by:
   a. $z \to iz$
   b. $z \to \left(\frac{1-i}{\sqrt{2}}\right) z$

c. $z \to (1 + i\sqrt{3})z$

d. $z \to \frac{z}{1+i}$

e. $z \to \frac{z+\bar{z}}{2}$

f. $z \to 1 - 2i + z$

g. $z \to \frac{z}{|z|}$

h. $z \to i\bar{z}$

i. $z \to -\bar{z}$

6. Find the sum of the following trigonometric series using de Moivre formula:

$$S_1 = \cos x + \cos 2x + \cdots + \cos nx = ?$$

$$S_2 = \sin x + \sin 2x + \cdots + \sin nx = ?$$

Geometry. Vectors.

Please, complete problems from the previous homework assignment. Review the classwork handout on vectors. Solve the following problems.

1. Using vectors, prove that the altitudes of an arbitrary triangle ABC are concurrent (cross at the same point H).
2. Using vectors, prove that the bisectors of an arbitrary triangle ABC are concurrent (cross at the same point O).
3. Using vectors, prove Ceva's theorem.
4. Let $ABCD$ be a square with side a. Point P satisfies the condition, $\overrightarrow{PA} + 3\overrightarrow{PB} + 3\overrightarrow{PC} + \overrightarrow{PD} = 0$. Find the distance between P and the centre of the square, O.
5. Let O and O' be the centroids (medians crossing points) of triangles ABC and $A'B'C'$, respectively. Prove that, $\overrightarrow{OO'} = \frac{1}{3}\left(\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'}\right)$.