Homework: Symmetries and Group Theory

May 2022

(1) (a) Show that the set of four elements $\{0, 1, 2, 3\}$ with the binary operation of addition modulo 4 is a group. (b) Construct the Cayley table of the group. (c) Show that it is a cyclic group. What is the generator of the group ? (d) Is the group abelian or non-abelian ?

(2) (a) Construct the Cayley table of the symmetry group of the equilateral triangle, the Dihedral group D_3. (b) Prove that it is a group. (c) What are the generators of the group ? (d) Is it abelian or non-abelian ?

(3) Construct all the subgroups of the Dihedral group D_3.

(4) Consider the set of roots of the equation $z^n = 1$ and multiplication as a binary operation. (a) Prove that it is a group. (b) Prove that this group is isomorphic to the group $\{0, 1, ..., n-1\}$ with a binary operation of addition modulo n.

(5) Consider the set S_n of permutations of n elements and the composition of two permutations as a binary operation. (a) Prove that S_n is a group. What is the order of the group. (b) Prove that the symmetry group D_n (Dihedral group) of a regular n-polygon is a subgroup of the permutation group S_n.