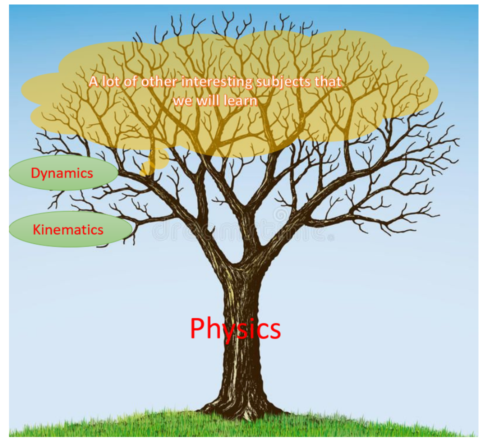
INERTIA AND NEWTON'S LAWS
NOVEMBER 16, 2021

Theory recap



Our last class was concluding our exploration of kinematics. Kinematics is a branch of physics that tells us how to describe the motion geometrically: in space and time. We have learned about distance, displacement, speed, velocity, acceleration and reference frames. But we did not ask the question: what caused the motion? In kinematics it was enough to know that motion occurs, and then we were able to relate some physical quantities to other physical quantities (like we related acceleration and time to change in velocity).

Today we start considering dynamics. Dynamics is another branch of physics and, as opposed to kinematics, it is concerned with reasons causing motion. These reasons are interactions with other objects.

First, let us ask a question, why should there be any interactions for motion to occur. Can't an object be in motion just by itself? It can, but only if it is already in motion. If the object does not interact with anything else, it will continue moving with the same velocity. In particular, if it was moving with zero velocity (which means it stayed at rest), it will continue to stay at rest in the absence of interactions with other objects. This is called **the law of inertia or Newton's first law** after Sir Isaac Newton, who has laid the foundation of classical mechanics which we still use after more than 350 years.

It makes perfect sense that if a body at rest does not interact with anything, it will stay at rest. It takes a little bit more effort to understand that the same principle applies to motion with constant velocity. Imagine a puck on very smooth ice. After being hit, it will go at almost constant velocity (only almost because there is still a tiny leftover friction). If the ice is even smoother, the velocity will change even less. We conclude that if there was no friction at all, the velocity would just stay constant indefinitely.

In slightly different words, Newton's first law tells us that in order to change its velocity, an object should interact with something else. In other words, some **force** should act upon it.

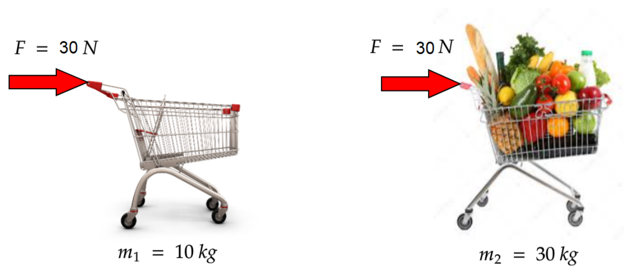
But Newton's first law does not tell us how will velocity change if there is some force acting on the object. To address this question we need **Newton's second law**: force $\vec{F}$ causes motion of an object of mass m with acceleration $\vec{a}$. These three quantities are connected by the following formula:

$$\vec{F} = m\vec{a}$$

So, forces cause acceleration, and for heavier objects acceleration produced by the same force is smaller. In this formula we encounter one familiar physical quantity - acceleration, and two new quantities - force and mass. Recall that acceleration tells us how velocity is changing with time and let us discuss force and mass now.

First, let's talk about mass. Although sometimes confused in everyday life, mass is not the same thing as weight. Weight is how much an object presses on the surface supporting it (and weight is really a force). It would be different in accelerating elevator or on other planets (we will come back to that in further classes). Mass, on the other hand, is a measure of inertia and is always the same. For any object mass could be calculated from Newton's second law by acting on it with a known force F and measuring acceleration a. Then $m = \dfrac{F}{a}$. In physics mass is measured in kilograms (kg) or in grams (g): 1 kg = 1000 g.

Now we move on to discuss force. A big part of our course will deal with different kinds of forces, some examples being: gravity force, normal force, friction force, elasticity force. What is common to all of them is that they describe how different bodies act upon each other. We will talk about them in detail in several next classes. Unit of force is called Newton and denoted by N. The force of 1 N acting on a body with mass 1 kg creates an acceleration 1 m/s^2, or $1\ \mathrm{N} = 1\ \dfrac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$.



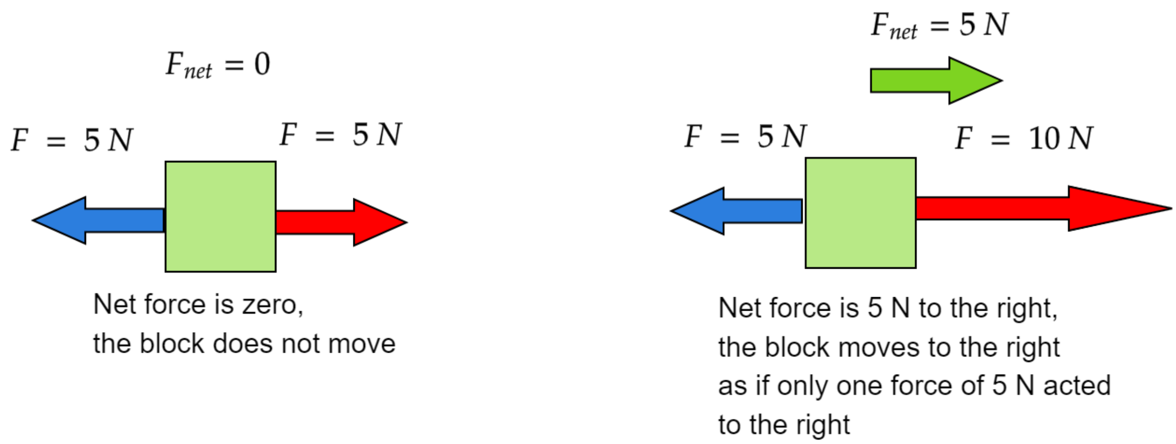
As an example we could consider a shopping cart. Suppose an empty shopping cart has a mass $m_1 = 10$ kg and after you load it with groceries for a week it has mass $m_2 = 30$ kg. If you push the cart with the same effort (same force), say 30 N, the heavier cart will have a smaller acceleration than the lighter cart. How many times smaller? Denote accelerations of the carts by a_1 and a_2 respectively. Let us use Newton's second law:

$$F = m_1 a_1 \Rightarrow a_1 = \frac{F}{m_1} = \frac{30\ \mathrm{N}}{10\ \mathrm{kg}} = 3\ \frac{\mathrm{N}}{\mathrm{kg}} = 3\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$F = m_2 a_2 \Rightarrow a_2 = \frac{F}{m_2} = \frac{30\ \mathrm{N}}{30\ \mathrm{kg}} = 1\ \frac{\mathrm{N}}{\mathrm{kg}} = 1\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

So a shopping cart three times heavier gets acceleration that is three times smaller.

Let me stress that force is a vector quantity (like velocity or acceleration): it has both magnitude and direction. Pushing a block to the east and to the north is not the same: in one case it will move to the east, and in the other case to the north. This is why both force and acceleration in Newton's second law have arrows on top: they are both vector quantities.



So far we have only discussed how does an object move when only one force acts upon it. What happens if there are several forces? It is not much more complicated: we have to add forces as vectors. For example, if you and your friend pull a block in the opposite directions with the same force 5 N, the block will not move: force vectors add up to zero. But if you were to pull stronger, say 10 N and your friend would still pull with 5 N, the block will move towards you as if a single force of 5 N was pulling it in your direction.

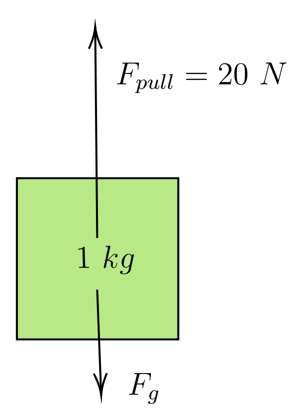
The first particular kind of force we will consider is gravity force. As we learned when discussing acceleration, in absence of air resistance all objects fall with the same acceleration, $g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$. In combination with Newton's second law this tells us that force due to gravity acting on a body of mass m should be

$$F_g = mg$$

Mass m in this equation is the same mass entering general Newton's second law. So actually, mass of an object determines not only how it will be accelerated by any force, but also what will be the gravity force acting on this object.

Let us consider one final example which concerns two forces one of which is the gravity force. Suppose we have a block with mass 1 kg which we pull upwards with force 20 N. Let us find its acceleration.

First thing which is always useful to do in problems of this kind is to draw a picture with all of the forces acting on the block. From it we could easily deduce what the net force is. Since the force of our pull and the gravitational force act in opposite directions, we have to subtract one from another. If we approximate free fall acceleration as $g = 10\ \frac{\mathrm{m}}{\mathrm{s}^2}$, gravitational force acting on the block is



$$F_g = mg = 1\ \mathrm{kg} \cdot 10\ \frac{\mathrm{m}}{\mathrm{s}^2} = 10\ \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = 10\ \mathrm{N}$$

So we pull stronger than gravity. Therefore the block will move up. In order to find its acceleration up we need to find the net force up, which is

$$F_{net} = F_{pull} - F_g = 20 \text{ N} - 10 \text{ N} = 10 \text{ N}$$

Finally, from Newton's second law we have the relation between net force and acceleration: $F_{net} = ma$ so we could find acceleration:

$$a = \frac{F_{net}}{m} = \frac{10 \text{ N}}{1 \text{ kg}} = 10 \ \frac{\text{m}}{\text{s}^2}$$

Homework

This time there are a bit more problems but they are mostly simple and conceptual.

1. Why do we need seatbelts in a car?

2. Many automobile passengers have suffered neck injuries when struck by cars from behind. How does the law of inertia apply here? How do headrests help to guard against this type of injury?

3. Two closed containers look the same, but one is packed with lead and the other with few feathers. How could you determine which had more mass if you and the containers are orbiting in a weightless condition in outer space?

4. You pull up a 2kg brick with a force of 30N. At a first moment the brick was at rest. Find the displacement of the brick in 3 seconds. (hint: find the acceleration first).

The problem below is a bonus problem.

***5.** A block is attached to the cart using four ropes, as shown in the picture. Force of tension in the horizontal ropes is $T_1 = 21\ N$ and $T_2 = 36\ N$, and in vertical ones - $T_3 = 30\ N$ and $T_4 = 60\ N$, free fall acceleration is $g = 10\frac{m}{s^2}$. What is the acceleration of the cart?