THE UNIVERSAL LAW OF GRAVITY

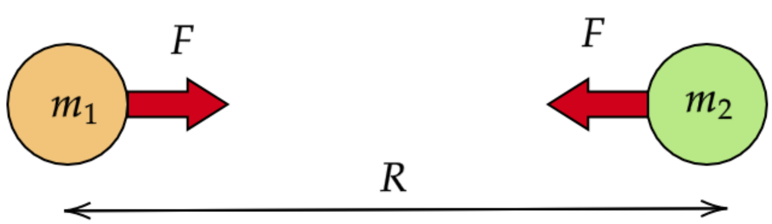
DECEMBER 21, 2021

Theory recap

We have already discussed that any object near the Earth is attracted to the Earth with the force mg where m is object's mass and g is the free fall acceleration. But this formula could only work near the surface of the Earth as the strength of gravity should decrease with distance. How does one take into account change of gravity with distance? Furthermore, we have discussed that different planets have different values of free fall acceleration. What does the free fall acceleration depend upon? Finally, is gravitational attraction somehow restricted to cosmic objects or does it act between any objects, no matter how small they are?

Answering these questions about gravitational force brings us to the Newton's universal law of gravity. Newton has discovered, that the force of gravity acts between any two objects and depends on the objects masses and the distance between them. The formula for the gravitational force between two (small) objects of masses m_1 and m_2 which are at the distance R from each other, is

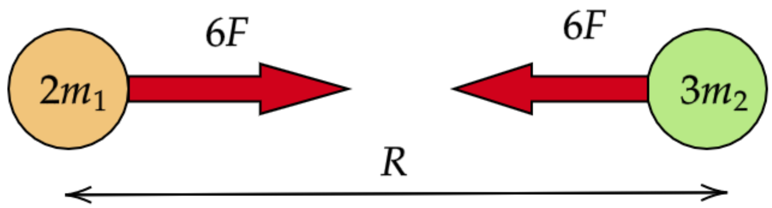
$$\boxed{F = \frac{Gm_1m_2}{R^2}} \tag{1}$$



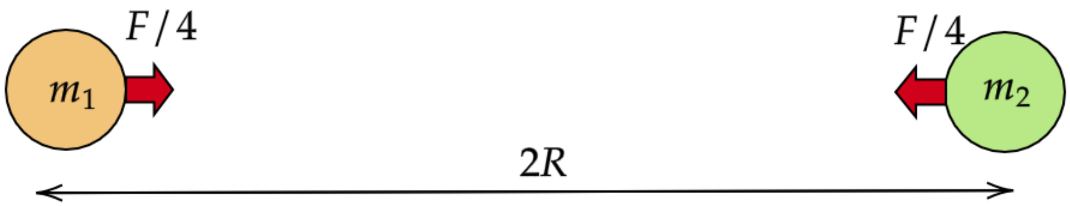
G in this formula is called **gravitational constant** and it is one of the fundamental constants of nature. It sounds amazing, but this formula describes the gravitational force between any two objects in the universe: it could tell us how gravity attracts stars to each other, or how the Earth pulls the Moon, or how the Earth pulls an ant, or how an ant pulls another ant! Any two objects experience gravitational attraction and G is absolutely the same for any of them. The numerical value of G is

$$G = 6.67 \cdot 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2}$$

Note that the formula (1) for gravitational force includes a product of masses, m_1m_2. Therefore, if we double mass of the first object and triple the mass of the second object, the gravitational force between them will increase $2 \cdot 3 = 6$ times (see figure below).

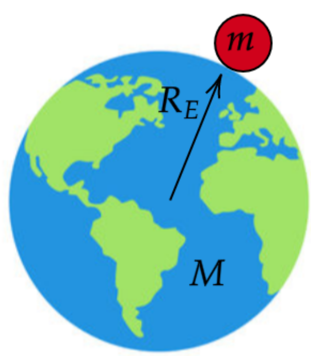


Gravitational force quickly decreases with distance between the objects. Distance squared in formula (1) tell us that if we put two objects two times father, their gravitational attraction will become four times weaker (see figure below).



Force is a vector quantity, so we need to specify the direction of gravitational force. Direction of gravitational force is such that the objects attract to each other. It means that gravitational force between two objects pulls these two objects in opposite directions (towards each other) in agreement with Newton's third law: action and reaction are equal and opposite.

Finally, let us understand what the universal law of gravity (1) tells us about the free fall acceleration. Consider a small object of mass m near the surface of the Earth. Let us call the Earth's radius R_E and the Earth's mass M.



We know that on one hand the force of gravitational attraction between the object m and the Earth is mg. On the other hand, we could use our formula (1) to calculate the same force in a different way. Let us note, that formula (1) as it stands is not directly applicable here: in formula (1) we assumed that the distance between the objects is much larger than any of their sizes. But there is a very similar formula which is valid for a spherically symmetric objects (and the Earth is spherically symmetric with a good accuracy) of any size. In that case we just need to use instead of R the distance between a small object m and the center of the Earth. If m is on the surface of the Earth, this distance is equal to the radius of the Earth R_E. Therefore, the force of attraction of the object m to the Earth is

$$F = \frac{GMm}{R_E^2} = mg$$

We have also explicitly written that the same force is usually expressed as mg. By canceling the common factor of m between the two expressions, we arrive to the expression for the free fall acceleration g via mass and radius of the Earth:

$$g = \frac{GM}{R_E^2}$$

The same formula would work for any planet: we see that free fall acceleration grows with mass of the planet but decreases with the size of the planet.

We could reverse the logic and use this formula to find mass of the Earth knowing its' free fall acceleration ($10\frac{\mathrm{m}}{\mathrm{s}^2}$), Earth's radius (6400 km) and universal gravitational constant ($6.67 \cdot 10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg}^2}$):

$$M = \frac{gR_E^2}{G} = \frac{10\frac{\mathrm{m}}{\mathrm{s}^2} \cdot (6.4 \cdot 10^6 \text{ m})^2}{6.67 \cdot 10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg}^2}} \approx 6 \cdot 10^{24} \text{ kg}$$

Homework

1. Find the gravitational force between two humans of mass 50 kg if the distance between them is 10 m.

2. Imagine that you are a NASA engineer in the early days of a Mars exploration program and you need to come up with a project of the first Mars rover. For this project it is critical to know the free fall acceleration on Mars, but no one has been there yet to measure it. Luckily, from astronomical measurements you know mass and radius of Mars. Mass of Mars is $M_M = 6.4 \cdot 10^{23}$ kg and radius of Mars is $R_M = 3400$ km. Use this information to calculate the free fall acceleration on Mars for success of the rover project.

***3.** Find the gravitational force acting on you from the Earth, the Moon and the Sun. Use Google search to find the necessary masses and distances (try to use a credible source).