IMPULSE AND MOMENTUM CHANGE

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Theory recap

To date we have discussed a lot of physics quantities : we learned about displacement and distance, velocity and speed, acceleration, mass, force and momentum. Let us briefly review the connections between these physical quantities.

We have learned that motion with constant acceleration is important. For such motion velocity changes with time as

$$\vec{v} = \vec{v}_0 + \vec{a}t \tag{1}$$

Force is related to acceleration by Newton's second law (for now let us assume that there is just one force acting on our body):

$$\vec{F} = m\vec{a} \tag{2}$$

And momentum is by definition the product of mass and velocity:

$$\vec{p} = m\vec{v} \tag{3}$$

On our last class we have already mentioned that in order to change momentum (without changing mass) one needs to change velocity, which requires acceleration. And by Newton's second law acceleration is provided by force. So to change momentum one needs to apply force. But what is the exact relation between force and change in momentum?

Let us set up the problem by saying that velocity of a block of mass m changed from initial value $\vec{v}_0$ to $\vec{v}$ because of force $\vec{F}$ acting on it for time t. Let us express the momentum in the final moment:

$$\vec{p} = m\vec{v} = m(\vec{v}_0 + \vec{a}t) = m\vec{v}_0 + m\vec{a}t = m\vec{v}_0 + \vec{F}t = \vec{p}_0 + \vec{F}t$$

In the above line of equations we used the relations (1), (2), (3). We also introduced the initial momentum $p_0 = mv_0$. Subtracting p_0 from both sides of the equation we get

$$\Delta\vec{p} = \vec{p} - \vec{p}_0 = \vec{F} \cdot t$$

This derivation tells us that change of momentum is equal to force multiplied by the time is takes to change the momentum this way. Product of force and time is called impulse:

$$\vec{J} = \vec{F} \cdot t$$

So we have seen that **change of momentum is equal to impulse**:

$$\boxed{\Delta\vec{p} = \vec{J}}$$

In order to change change momentum force and time are both equally important. We could act for a short time with a large force, or for a long time with a small force and get the same change in momentum. If a car stops by hitting the brakes, it takes at least several seconds to stop. If the same car hits a wall, it stops almost instantaneously. But momentum change is the same, which means the force is much larger in the case of hitting the wall. This large stopping force causes damage which was absent in the case of braking.

Let us note that impulse is a vector which has the same direction as force. This makes sense because momentum is also a vector and momentum change is a vector as well.

If there are several forces acting on some object, we could calculate their impulses one by one and add them up in order to get the resulting momentum change. Alternatively, if these forces are always acting together, we could first calculate the net force and then multiply it by time to get the impulse.

Let us use impulse to justify momentum conservation law which we stated on our last class. Suppose we have two blocks and block 1 acts with a force F on block 2. During some time t change of momentum of block 1 due to this internal force is $\Delta p_1 = Ft$. By Newton's third law block 2 acts on block 1 with force $-F$. Correspondingly during the same time t change of momentum of block 2 due to this internal force is $\Delta p_2 = -Ft$. If there are no other (external) forces, this is the only change in momentum of blocks. Therefore the change of total momentum is

$$\Delta p_{tot} = \Delta p_1 + \Delta p_2 = Ft - Ft = 0$$

So we see that total momentum change is zero, which means that it is conserved.

Homework

1. A rubber ball of mass 10 g is dropped down from the top of 180 m building. The ball hits the ground and bounces up with the same speed. Find the force applied by the ball to the ground if the collision time is 0.01 s.

2. A soccer ball with mass 400 g moves at a speed of 25 m/s. If the ball hits the chest of the goalkeeper it bounces back with the same speed and the collision time is 0.025 s. If the goalkeeper catches the ball with his (her) hands, the speed of the ball becomes zero in 0.04 s. Find the force applied by the ball to the goalkeeper in both cases.

***3.** A block initially at rest is located on a horizontal plane. It is being pushed horizontally with a force F for time Δt after which this force disappears. What is the friction force acting on this block if it stops in $3\Delta t$ after force F disappeared?