POTENTIAL ENERGY
FEBRUARY 1, 2022

Theory recap

Last time we discussed kinetic energy. We learned that any moving object with mass m and speed v has kinetic energy equal to $E_{kin} = \frac{mv^2}{2}$. But energy also exists in other forms, because we know situations when kinetic energy alone is not conserved. For instance, consider an object in free fall. Its speed is constantly increasing, so kinetic energy gets larger and larger. Where does the object get its kinetic energy from?

This question brings us to the concept of potential energy. The name suggests that an object with potential energy has a potential to gaining kinetic energy or transferring its energy to other objects. It is clear that the higher some object was initially, the greater will be its speed near the ground. Therefore potential energy should be bigger when the object is higher. The formula for potential energy is:

$$E_{potential} = mgh$$

Here m is mass of the object, g is free fall acceleration and h is the height above ground (we will return a bit later to the question if ground level is really important). So we see that heavier objects have larger potential energy and it also grows with height. It is very important that potential energy only depends on the current position of the object and does not depend on how the object got there.

Consider a book lying on a desk. If the book is moved to the floor, its' potential energy is changed. For instance, if the book has mass 0.9 kilograms and the desk is 0.8 meters high, we could find the change in the potential energy of the book. To find the change, we need to know its initial and final values. Initially, the book is 0.8 meters above the floor, so its' potential energy is

$$E_{pot,1} = mgh = 0.9 \cdot 10 \cdot 0.8\text{J} = 7.2\text{J}$$

At the final point the book is on the floor, in other words it has elevation 0 above the floor. So its potential energy at this moment is

$$E_{pot,2} = 0$$

The change in potential energy is

$$\Delta E_{pot} = E_{pot,2} - E_{pot,1} = 0 - 7.2\text{J} = -7.2\text{J}$$

But why did we choose to measure the elevation with respect to the floor? Why don't we use something more universal, like the surface of the ground? Let's say the desk it at the second floor which is 3 meters above the ground level. With respect to the ground level when the book is on the desk, it has potential energy

$$E_{pot,1} = 0.9 \cdot 10 \cdot 3.8\text{J} = 34.2\text{J}$$

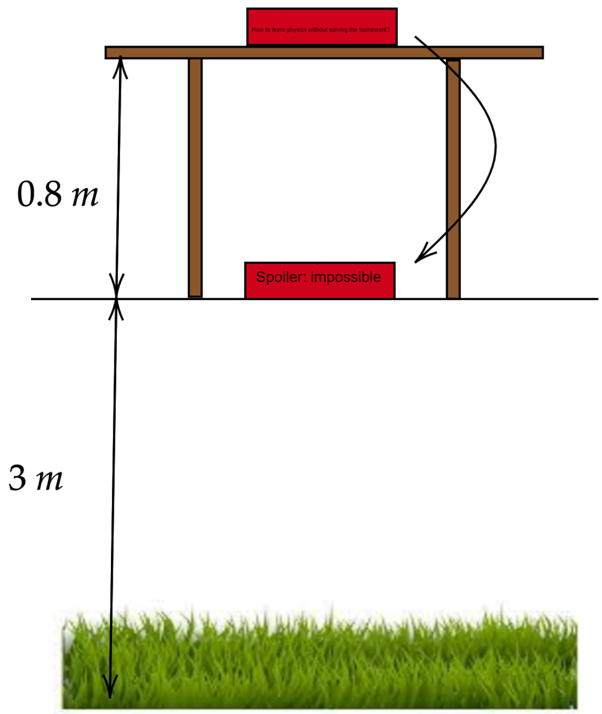
and when the book is on the floor, it has potential energy

$$E_{pot,2} = 0.9 \cdot 10 \cdot 3\text{J} = 27\text{J}$$

The change in potential energy is

$$\Delta E = 27\text{J} - 34.2\text{J} = -7.2\text{J}$$

again! The lesson here is that even though we need to choose a **reference point** to define potential energy (with respect to what we are measuring elevation), the choice of the reference point does not influence the change in potential energy during some process. And in fact it is only the change in potential energy that we will always be interested in when writing down energy conservation.



So we could choose the reference point to be the most convenient point in any particular problem.

We can introduce the concept of **total mechanical energy**, which is the sum of kinetic and potential energies:

$$E_{mechanical} = E_{kinetic} + E_{potential} = \frac{mv^2}{2} + mgh$$

Now we could state the law of **conservation of mechanical energy**. It applies in absence of certain forces, such as friction (we will have to wait until the next time for more details) and states that the total mechanical energy is conserved, so it does not change with time:

$$E_{mechanical} = const$$

For instance, we could see how energy conservation really works for an object in free fall. Assume that an object starts falling from rest at height h above the ground. With what speed v it will hit the ground if we could neglect air resistance?

Let us at energy conservation. During the fall, potential energy is transferred into kinetic energy, but their sum (total mechanical energy) is conserved. So total mechanical energy at the beginning of the fall must be equal to the total mechanical energy near the ground. At the beginning kinetic energy was zero, as or object had no speed, but potential energy was mgh, so:

$$E_{mech,1} = E_{kin,1} + E_{pot,1} = 0 + mgh = mgh$$

On the other hand, near the ground kinetic energy is $\frac{mv^2}{2}$ where v is speed of the object there. Potential energy is now zero, because the object is at zero height. Therefore

$$E_{mech,2} = E_{kin,2} + E_{pot,2} = \frac{mv^2}{2} + 0 = \frac{mv^2}{2}$$

So, energy conservation law tells us that

$$E_{mech,1} = E_{mech,2} \Longrightarrow mgh = \frac{mv^2}{2}$$

We can cancel the mass in this relation and use it to solve for v:

$$v^2 = 2gh \Longrightarrow v = \sqrt{2gh}$$

Homework

1. A 10 g bullet is sent up at a speed of 300 $\frac{m}{s}$. How high it will go? Solve this problem in two ways (through kinematics and through energy conservation).
2. A 50 g ball is falling down. As the ball passes a certain distance its potential energy changes by 2 J. Calculate this distance. Does this distance depend on the initial velocity of the ball?
3. In the class we watched videos comparing how two different balls bounce off a desk. A black rubber ball has mass 43 grams while a tennis ball has mass 56 grams. They are both initially held at 60 cm above the desk and then released with zero initial speed. After the first bounce the maximal height reached by the black rubber ball was 45 cm, while for the tennis ball it was 35 cm. Compare how much energy each of the balls lost during the first collision. Find what **fraction of initial kinetic energy** was lost during the first collision for each of the balls.
4. Potential energy could be used for energy storage (and then at a necessary moment converted into electricity). A prominent example of this idea is a water power plant where a huge amount of water is stored in an elevated reservoir. The largest US power plant is Grand Coulee in Washington. It contains 10 cubic kilometers of water elevated at a height of approximately 100 meters. Calculate, how much energy in joules is stored in the Grand Coulee power plant. **Bonus question**: ask your parents about your house average electric energy consumption in a year. Then estimate for how many years the reservoir of the Grand Coulee power plant could supply your house with electric power, if all of its' potential energy could be converted into electricity without losses.

Hint: you may need some unit conversions for this problem. One cubic meter of water has mass 1000 kilograms (think carefully, how many cubic meters are in one cubic kilometer). A common unit of electric energy is kilowatt-hour (kWh), 1 kWh = $3.6 \cdot 10^6$ *J.*

***5.** During the class we only discussed potential energy of small objects, for which their own size is much smaller than their elevation. In this problem you need to think one could generalize the concept of potential energy without the limitation that an object is small. Find potential energy of a thin rod of mass m and length l which is placed vertically on the floor (height is measured from the floor).