GAS LAWS CONTINUED

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Theory recap

Boyle's law recap. Last time we discussed Boyle's law. We learned that at constant temperature increasing pressure of a gas leads to decreasing volume, according to the formula:

$$\frac{p_2}{p_1} = \frac{V_1}{V_2}.$$

An equivalent way of saying it is that the product of pressure and volume is constant when temperature is constant:

$$pV = const \quad \text{for} \quad T = const$$

There is an intuitive explanation of why Boyle's law is valid. As we discussed, pressure is produced by gas atoms or molecules colliding with the walls of the container. There are lots and lots of the collisions every second and on average they lead to some particular pressure. Imagine that we decrease the volume of the gas. Then is becomes denser and collisions of gas particles with the walls happen more often. Each of these collisions still produces the same force on the wall as before (because this force depends on the speed of a gas molecule which has not changed since temperature is constant). But since there are more collisions, the total force exerted on the walls of the container is larger and therefore pressure is larger. This is basically how Boyle's law works.

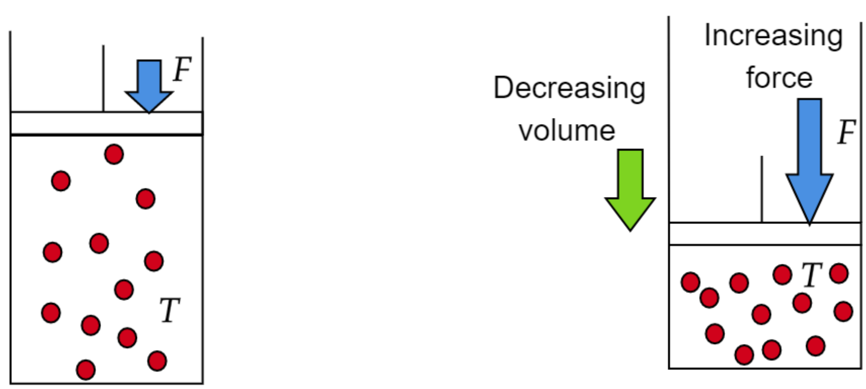

FIGURE 1. **Left figure:** Initially gas has some volume. Molecules collide with the piston and produce some force. An equal force is needed to balance it. **Right figure:** Volume of gas is decreased. Molecules are packed more densely and collide with the piston more often leading to the larger force. So, pressure of the gas increases.

Gay-Lussac's law. Today we will further learn what happens if we change temperature. As we discussed, there are two other gas laws in each of them one of the two remaining quantities (pressure or volume) is kept fixed. Let us begin with fixed volume.

For a fixed volume we can imagine a sealed container without a piston. Since we want to keep track of pressure, a pressure sensor can be inserted in this container. In order to change the temperature of the gas we can put the whole container into hot water or some other environment of a given temperature. If we wait a bit, the container and the gas will reach the same temperature as the environment. Then we measure temperature pf the environment and assume that gas has the same temperature.

Assume that our gas in the closed container initially has pressure p_1 and temperature T_1 (in Kelvins). Then we increase its temperature to T_2 (also in Kelvins). French physicist Joseph Louis Gay-Lussac has discovered that pressure then also increases to p_2 according to the following relation:

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}.$$

An equivalent way of writing this relation is:

$$\frac{p}{T} = const \text{ for } V = const.$$

It is called Gay-Lussac's law.

For example, if at 300 K a gas has pressure 300 kPa then at 400 K and at the same volume it will have pressure 400 kPa, at 500 K pressure will become 500 kPa etc.

Why is it important that temperature in Gay-Lussac's law is measured in Kelvins? Imagine that instead of Kelvins we measured temperature in Celsius. As we know,

$$T = t + 273$$

where T is temperature in Kelvins and t is temperature in Celsius. So if we want to express temperature in Celsius in Gay-Lussac's law we would get

$$\frac{p_2}{p_1} = \frac{t_2 + 273}{t_1 + 273}$$

Does not look so nice, does it? We see that Kelvin scale allows us to write Gay-Lussac's law in a much simpler form.

Let us discuss a microscopic reason behind Gay-Lussac's law? Remember that temperature is a measure of average kinetic energy of the molecules. If we increase temperature, molecules have more kinetic energy. As a result, each individual collision of a molecule with the wall produces a larger force. If the volume stays the same the rate of collisions also grows because of increased speed of the molecules. Since there are more collisions per unit time and each collision is stronger, the total force increases. This means that pressure increases with growing temperature and constant volume. This is what Gay-Lussac's law tells us.

Homework

1. A cylinder is filled with gas. The pressure inside is 10000 Pa, the temperature is 20°C. We increase the temperature to 100°C. What happens to the pressure inside the cylinder? Calculate the new pressure.

2. A gas has initial pressure 100 kPa, volume 100 cm^3 and temperature 27 °C. First, the gas is compressed at constant temperature so that its volume decreases two times. Then the volume of the container is fixed and the container is cooled down to -123 °C using liquid nitrogen. Find the final pressure of the gas.

***3.** Pressure of air in a bottle at 7°C is equal to the atmospheric pressure 100 kPa. How much does one need to heat the bottle so that a cork closing the bottle will be pushed out? Without heating the cork could be pulled out by force 10 N. Cross-section area of the cork is 2 cm^2.