PRESSURE IN LIQUIDS.

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Theory recap

Today we start discussing the physics of liquids. We will be interested in equilibrium conditions in a fluid, so we will consider a stationary liquid, which does not move or flow. At every point liquid has some pressure, similarly to a gas. However, in a liquid pressure varies with height more significantly than in a gas (as we will see below it is because liquid is much more dense than gas).

Consider a cylindrical vessel with liquid. Let the cross section area of the vessel be A, height of the liquid level be h, and density (mass per unit volume) of the liquid be ρ. Further let us assume that there is no atmospheric pressure (we will add it to consideration a bit later). We are interested in finding the pressure of the liquid at the bottom of the vessel. In order to do it let us consider all forces acting on the liquid. There are two forces: gravity force mg and normal force from the bottom of the container N. Mass of the liquid is

$$m = \rho V = \rho A h$$

where V is volume of liquid, equal to product of cross section area and height. Normal force from the bottom is related to the pressure at the bottom, since pressure is the force per area:

$$p = \frac{N}{A} \implies N = pA.$$

Because liquid is in equilibrium, the two forces must balance each other:

$$N = mg \implies pA = \rho A h g \implies p = \rho g h.$$

The last formula is what we were looking for: it tells us how pressure grows with depth. For larger densities pressure grows faster. The cross section area canceled in the expression for pressure, so the formula is valid for a vessel of any size (and, in fact, any shape).

What changes about this derivation if there is also atmospheric pressure p_0? It means that pressure of liquid in the very top layer is now not 0, but p_0. However, the difference in pressure between the top and the bottom happens still arises because of the weight of the fluid. Therefore, the pressure at the bottom level is

$$p(h) = p_0 + \rho g h.$$

As an example let us ask at what depth in a lake (or a sea) the additional pressure created by water is equal to normal atmospheric pressure, about 100 kPa. Density of water is about 1000 kg/m^3. We need to find h such that

$$\rho g h = p_0,$$

therefore

$$h = \frac{p_0}{\rho g} = \frac{100,000 \text{ Pa}}{1000 \text{ kg/m}^3 \cdot 10 \text{ N/kg}} = 10 \text{ m}.$$

So 10 meters of water create pressure about one atmospheric pressure. Similarly, 20 meters of water create twice the atmospheric pressure and so on. This pressure is very important to consider for scuba divers and submarines.

Homework

1. Density of mercury is approximately 13,500 kg/m^3. What height of a mercury column creates normal atmospheric pressure?
2. This problem will demonstrate why we did not bother with variation of pressure with height when discussing gas laws. Find the pressure variation between the top and the bottom of a container with nitrogen N_2 at a temperature 27°C and an average pressure 100000 kPa (one atmosphere). The container is 1 m high. How many times smaller is the pressure variation compared to the average pressure? *Hint: you would need to know the gas density. Use the definition of density (mass over volume) and the ideal gas equation of state. Do not forget how to relate the number of moles to mass.*