Homework 6

**Superposition principle.**

The beauty and convenience of the concept of electric potential is that using the electrical potential we can easily calculate the potential energy of a charged object in the electric field created by arbitrary configuration of other charged objects.

Let us consider the following example.

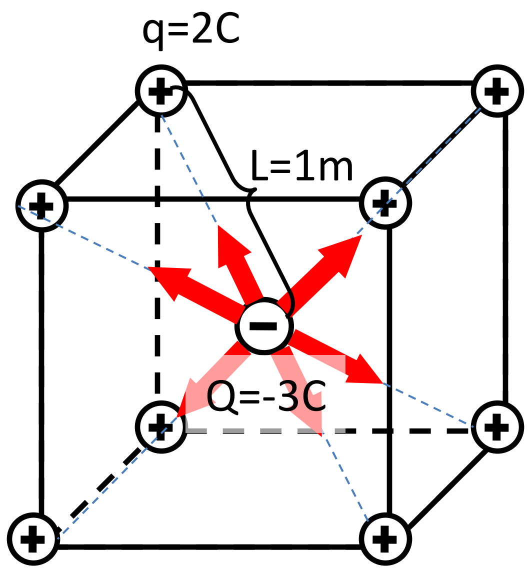


Figure 1. The red arrows show the electrostatic forces applied to the negative charge.

**Given:**

Eight point charges q=2C each are placed in the corners (vertices) of a cube (see Figure 1 above). The positions of the positive charges are fixed. The distance between the center and a corner of the cube is 1m. A negative charge of Q=3C is placed in the center of the cube.

**Find electrostatic potential energy of the negative charge.**

**Solution:**

As we remember, a possible way to calculate the electrostatic potential energy of a charge in a certain point is to calculate the electrostatic potential in this point and multiply it by the charge. Let us calculate the electrostatic potential in the center of the cube (black point in the Figure 2.

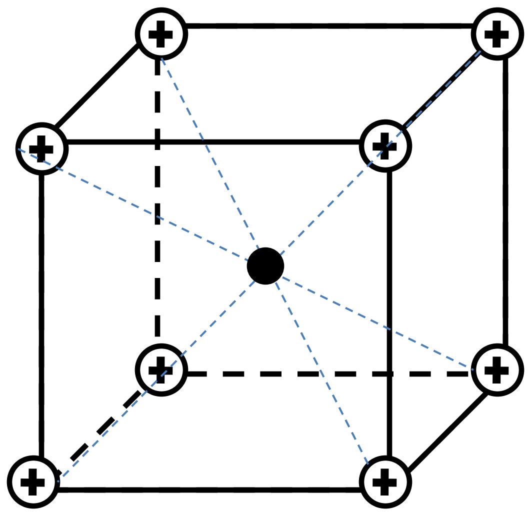

Figure 2.

At a first glance the problem looks difficult. But, in fact, it is not. To solve it, we will use the ***principle of superposition.*** According to this principle, we can calculate the potentials created in the center of the cube by each of the positive charges separately. After that we will just add these potentials together.

a) Let us pick just one positive charge (Figure 3).

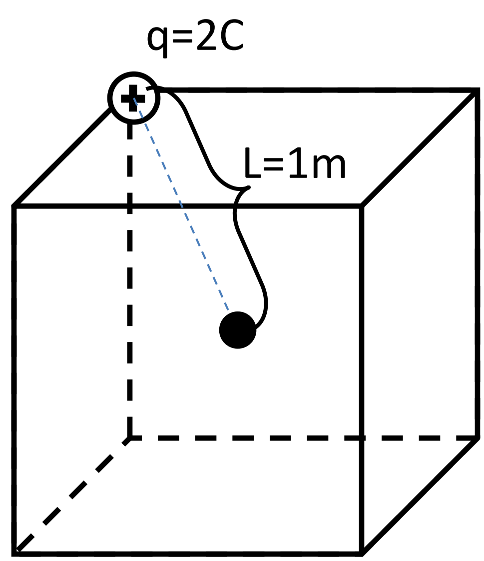


Figure 3.

Then let us calculate the electrostatic potential created by this charge at the center of the cube.

$$\boldsymbol{\varphi_{one\ charge}} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{L}} \approx 8.9 \cdot 10^9 \left(\frac{N \cdot m^2}{C^2}\right) \cdot \frac{2(C)}{1(m)}$$
$$\approx 1.78 \cdot 10^8 (V)$$

b) Now, we have to take another positive charge, calculate its contribution to the potential etc. But, there is a simpler way. We can use the ***symmetry principle*** which we discussed earlier. As long as all the corners of the cube occupy equivalent positions with respect to the center of the cube, there is no reason to prefer one corner to another. Thus, the contributions of the equal positive charges placed in the corners of the cube to the potential in the cube's center should be equal. So we can just multiply the contribution of one positive charge by 8 – the number of corners.

$$\boldsymbol{\varphi_{total}} = \boldsymbol{\varphi_{one\ charge}} \cdot 8 = 1.78 \cdot 10^8 (V) \cdot 8$$
$$\approx 1.42 \cdot 10^7\ (V)$$

c) Now we can easily calculate the potential energy ***P*** of the charge ***Q=-3C*** placed in the center of the cube:

$$\boldsymbol{P} = \boldsymbol{\varphi_{total}} \cdot \boldsymbol{Q} = 1.42 \cdot 10^7\ (V) \cdot (-3)(C) = -4.26 \cdot 10^7 J$$

What happens if the charges at the "bottom" vertices are negative (Figure 3)?

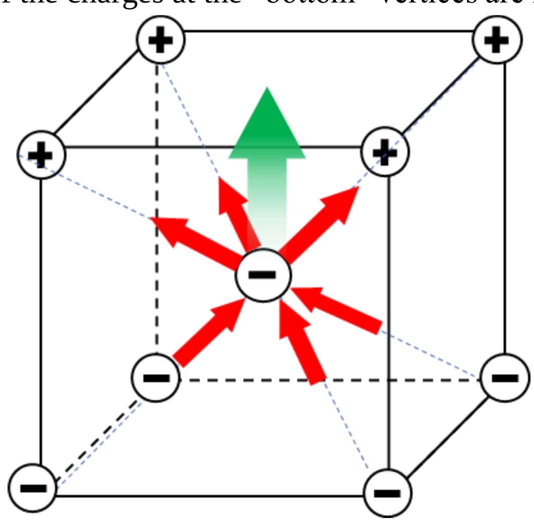

Figure 3.

Now, as we can clearly see the negative charge in the center will be pushed up, since it is repelled from the bottom and attracted to the top. So, if we let it go, it will be accelerated and, when reaches, say, the middle of the top facet it will have some kinetic energy. Based on this, we can conclude that our charge has some potential energy while placed in the center of the cube. But let us calculate this potential energy using our recipe: calculate the potential and multiply it by the charge (-3C). The contribution of each of the top positive charges we have calculated earlier: $1.78 \times 10^8$ V. We have to multiply it by 4, since there are 4 charges on the top vertices). The contribution of each of the bottom negative charges is negative: $-1.78 \times 10^8$ V. We have to multiply it by 4 as well and add the positive contribution of the top charges. The result is zero! And it is correct! But if we multiply -3C by zero to find the potential energy we will get zero. No potential energy!

To resolve this paradox let us remember that not the absolute potential energy, but the change of potential energy is physically meaningful. Potential energy in the center is zero, but it is not ***minimal*** potential energy. If you will calculate the potential energy of our charge in the center of the upper facet, you will have a negative value. So, change of the potential energy will be positive and will be equal to the kinetic energy acquired by our charge.

Problems

1. Assume that the bottom charges of our cube are negative (Figure3) and each of them is equal to -2C. Charges in the upper corners are positive and equal to 2C each. Find potential energy of our -3C charge in the middle of the upper facet.

2. Find potential in the center of a uniformly charged thin sphere having a total charge of 1C and radius 1m.