Work and Kinetic Energy

2nd Newton's Law can be rewritten as:

$$m\Delta v = F\Delta t$$

$$mv\Delta v = F(v\Delta t)$$

One can show that the left-hand-side is a change in Kinetic energy, $K = \frac{mv^2}{2}$

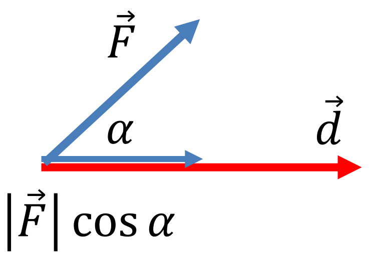
The right hand side (Force times Displacement) is called work, $W = F\Delta x$

This leads to a very important result:

$$\Delta K = W$$

A more general definition of work when Force and Displacement are given as vectors $\vec{F}$ and $\vec{d}$ (in 2D or 3D case):

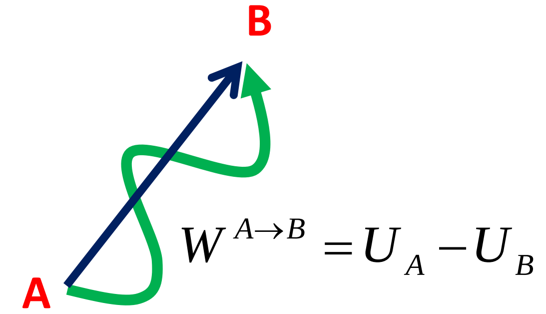
$$W = |\vec{F}||\vec{d}|\cos\alpha$$



Conservation of Energy

Work by forces such as gravity of electrostatics depend only on initial and finite point, but not on the path itself. These forces are called Conservative (or Potential). For them we can introduce Potential energy, U:

$$W^{A\to B}_{conservative} = U_A - U_B = -\Delta U$$



$$\Delta K = -\Delta U + W_{non-conservative}$$

Therefore, if there is no non-conservative forces (no friction, engine or other external force), Total Energy (Kinetic + Potential) is conserved:

$$E = K + U = const$$

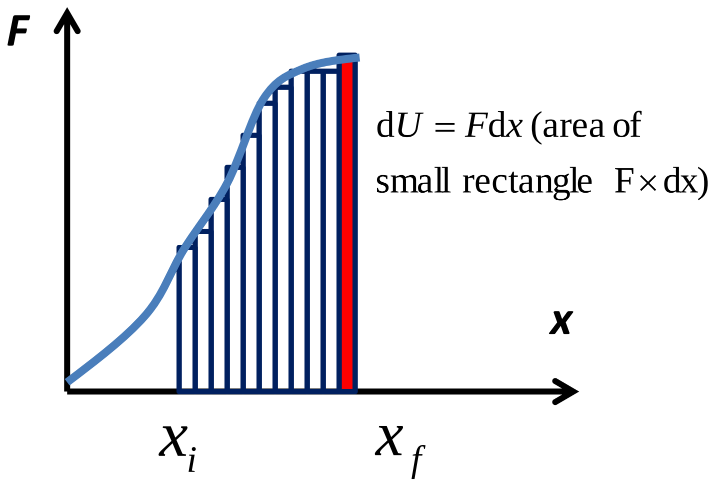
More generally, its change is equal to work of non-conservative forces:

$$\Delta E = W_{non-conservative}$$

Potential Energy = Integral of Force

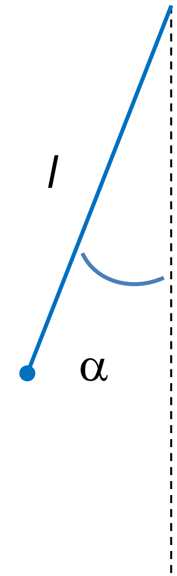
Let us calculate the work done ***against*** the a force.
This work is stored as potential energy:

$$\Delta U = W = \int_{x_i}^{x_f} F(x)\,\mathrm{d}x$$



Type of force	F	U
Gravity (on Earth surface)	mg	mgh
Hooke's Law (spring force)	kx	$\frac{kx^2}{2}$
Newton's Law of Gravity	$F = \frac{Gm_1m_2}{r^2}$	$U = -\frac{Gm_1m_2}{r}$

Homework



Problem 1

A pendulum is made of a ball attached to a weightless string of length *l*. The pendulum has been deflected by angle α from its stable vertical orientation (see the Figure). Find the maximum speed of the ball after the pendulum is released. Neglect air resistance.

Problem 2

A bobsleigh goes down the track whose initial point is at height ***h=150 m***. If there were no friction and no air resistance during the descend, find the distance ***d*** that bobsleigh had to travel after the finish line, before coming to a complete stop. Coefficient of kinetic friction on that horizontal part of the trip is ***μ=0.5***.