Homework 3.

Reflection at a spherical surface.

During last class we discussed reflection at a spherical surface. I would say that this is a cornerstone problem in geometrical optics. To warm up ☺ I would like to consider reflection at a plane mirror (Figure 1). The physical laws we will use are:

- In a uniform media light propagates along a straight line
- The reflection angle is equal to the incidence angle.

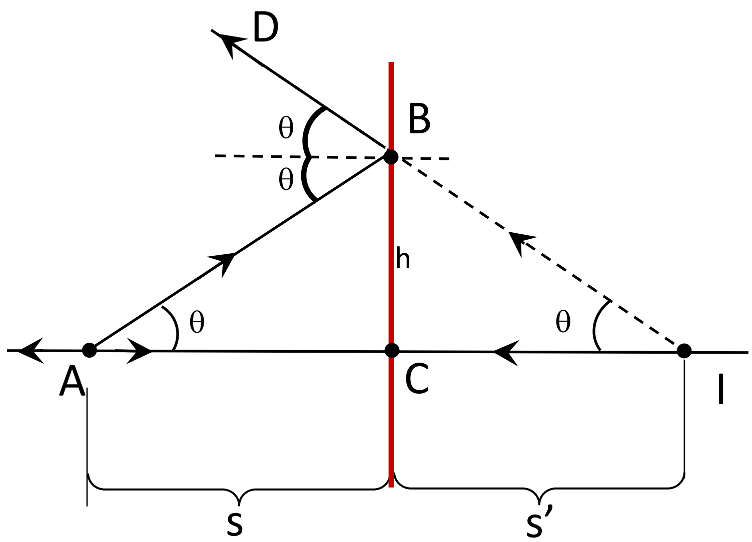

Figure 1. Reflection at a plane mirror.

Let us place a pint light source in point A. Let us further consider two rays of light emitted by the source A. Of course there are infinitely many rays emitted by A, but we will consider just two. First ray, AC, is emitted perpendicularly to the mirror plane (Figure 1, red) and is reflected back. The second, AB, hits the mirror plane at an angle $\boldsymbol{\theta}$ and reflects under same angle. The reflected ray is BD. Let us extend the lines AC and BD to their intersection point I. If you are looking at the mirror from the left, you see two diverging rays of light CA and BD. It looks for you that the rays are emitted from point I, which is the image of the source A. As long as the reflected rays do not intersect in reality, they just diverge as if they would have been emitted from one point, the image is called “virtual”. It is easy to see that triangle ABC is equal to the triangle CBI and the distance ***s*** from the object to the mirror is equal to the distance ***s'*** from the mirror to the image.

Now, let us go to the spherical mirror (Figure 2). Again, point source of light A is placed at a distance ***s*** from the mirror. Point O is the center of the mirror sphere, OB=***r*** is its radius of curvature. The center of curvature O and the light source A are at the opposite sides of the mirror.

We will call such mirror as “convex”. Let us assume that ***r*** is much larger than the diameter of the mirror ***d*** (Figure 2), using other words, the mirror is ***almost*** flat (Figure 2).

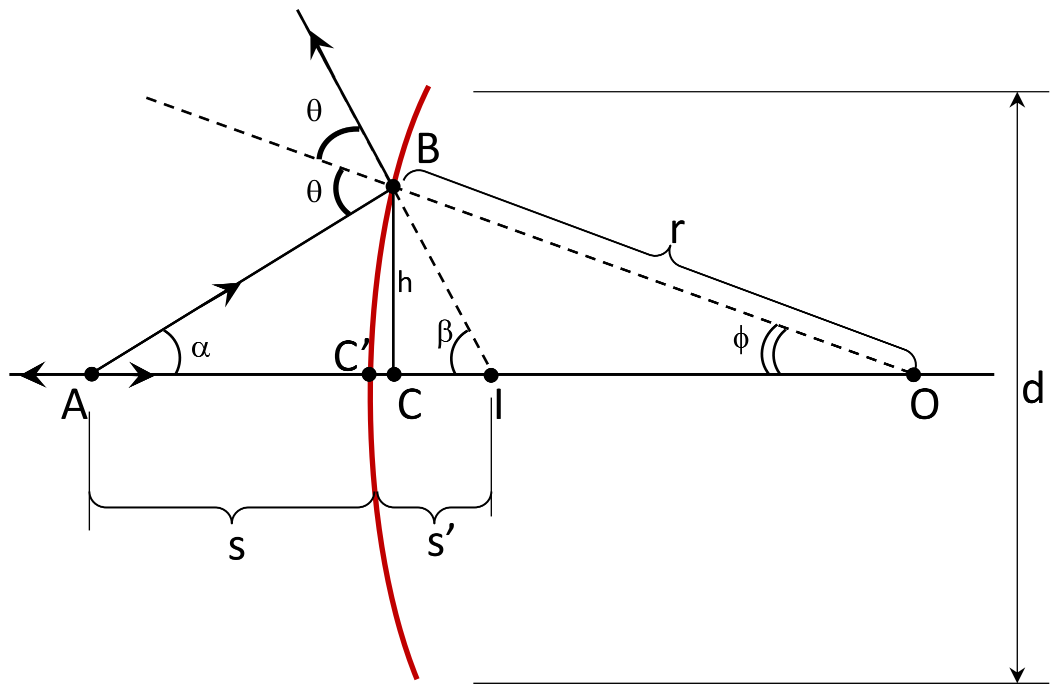

Figure 2. Reflection at a spherical surface.

Ray AB reflects from the surface (from the tangent plane to the surface in the point of reflection) and becomes ray BD. The radius OB is perpendicular to the reflection plane (radius is always perpendicular to the tangent plane in the point of intersection of the radius and the sphere). Let us draw perpendicular to the line AO from point B - line BC. The length of segment BC is ***h***. We assumed that the surface is almost flat, so ***we neglect the distance between C’ and C and consider these two points as one.*** Since angle θ is exterior tho triangle ABO we can write:

$$\alpha+\phi=\theta \quad (1).$$

For triangle ABI we can write

$$\alpha+\beta=2\theta \quad (2),$$

since 2q is an exterior angle for this triangle. From expressions (1) and (2) we can obtain:

$$\alpha-\beta=-2\phi \quad (3).$$

Then let us remember that we assumed that mirror is *almost* flat. The $\boldsymbol{h}$ cannot exceed the diameter of the mirror $\boldsymbol{d}$. This means $\boldsymbol{r}>>\boldsymbol{h}$. Let us further assume that the distances $\boldsymbol{s}$ and $\boldsymbol{s'}$ are also much higher than $\boldsymbol{h}$, so all the angles $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\phi}$ are small. For a small angle a the following expressions are valid:

$$sin\alpha \approx tg\alpha \approx \alpha \qquad (4).$$

$$cos\alpha \approx 1 \qquad (5).$$

Using (4) and (5) we can write:

$$\alpha \approx \frac{h}{s}; \quad \beta \approx \frac{h}{s\prime}; \quad \phi \approx \frac{h}{r} \qquad (6).$$

Expression (3) now can be written as:

$$\frac{h}{s} - \frac{h}{s\prime} = -2\frac{h}{r} \quad (7).$$

After dividing both parts of the expression (7) to h we have:

$$\frac{1}{s} - \frac{1}{s\prime} = -2\frac{1}{r} \quad (8).$$

Formula (8) connects distance to the object, distance to the image and radius of curvature of the reflecting spherical surface. If we take $r \to \infty$ the spherical mirror becomes flat and, as expected, from formula (8) we have: $\boldsymbol{s}=\boldsymbol{s'}$. As the flat mirror, a convex mirror produce virtual image.

If we repeat the procedure with the concave mirror, expression (8) will transform to

$$\frac{1}{s} + \frac{1}{s\prime} = 2\frac{1}{r} \qquad (9).$$

It is possible to unify expressions (8) and (9) into one general expression and use sign rule given below.

$$\frac{1}{s} + \frac{1}{s\prime} = -2\frac{h}{r} \quad (10).$$

Sign rules:

1. The object distance s is positive if the object is to the left of the mirror (real object).
2. The image distance s’ is positive when the image is to the left of the mirror (real image) and negative when the image is to the right of the mirror (virtual image).

3. The radius of curvature r is positive when the center of curvature is to the right of the mirror surface (convex mirror) and negative when the center of curvature is to the left of the mirror surface (concave mirror).

Let us move object far away from the convex mirror ($s \rightarrow \infty$). In this case $\alpha \rightarrow 0$, so the rays from the object are parallel to the mirror axes. As we can see from the equation (8)

$$s' = \frac{r}{2} \quad (11)$$

In case we send the light rays to the spherical mirror parallel to the mirror axes, the reflected light will diverge as if they would have come from the point which is at a distance r/2 from the mirror surface. This distance is called "focal length".

Problem

1. Make a detailed picture and prove the expression (9) for a concave mirror. What kind of image we will have in this case?
2. The Moon is reflecting from the surface of a big lake. Find the distance from the surface of the lake to the Moon's reflection. Make the picture. (take the distance from Earth to Moon as 380,000 km).