Homework 6

Thin lenses combined. Refractive (optical) power.

Let us consider the image produced by 2 closely spaced biconvex thin lenses. Let us also assume that the spacing between the lenses is much less than focal distances of the lenses. The corresponding ray diagram is shown in Figure 1.

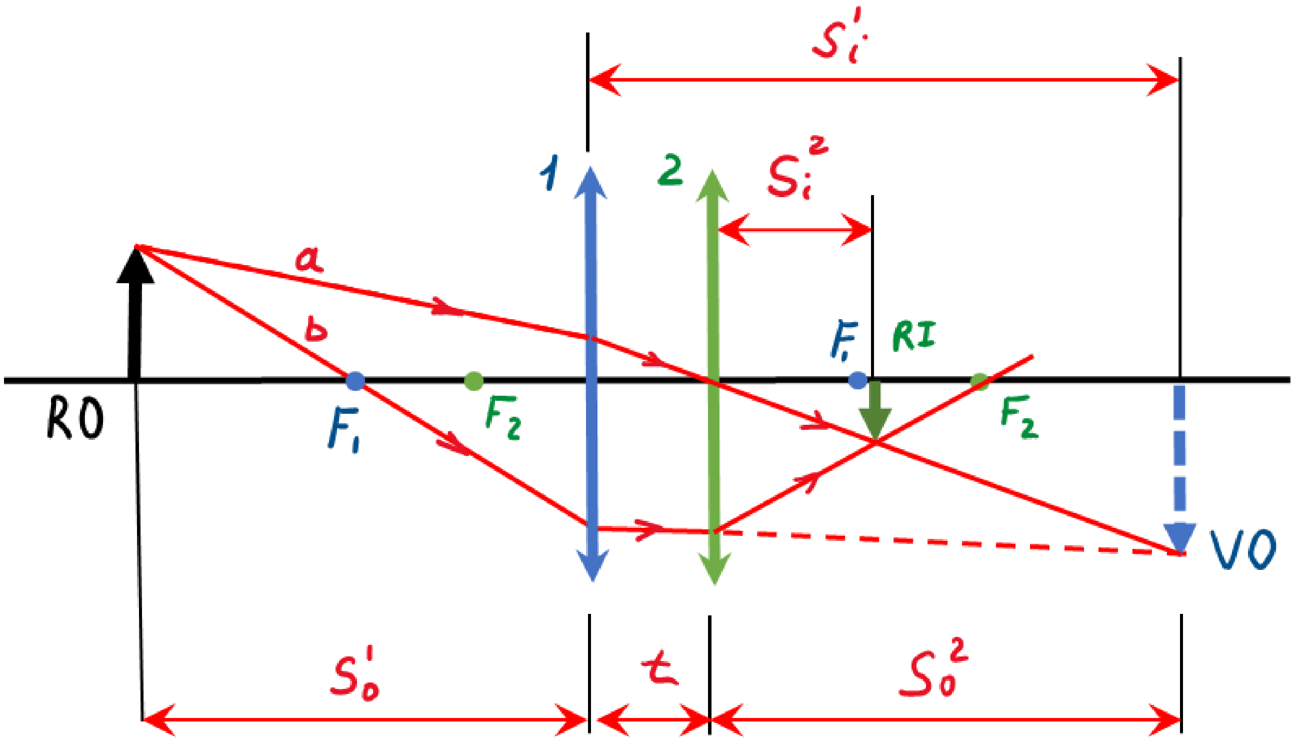

Figure 1. Image formed by two closely spaced thin convex lenses

Let us consider just two rays propagating from the object RO: ray **a** and ray **b**. Both rays are refracted by lens 1 (a blue one) an would have crossed at the tip of the arrow VO (virtual object). But both rays are intercepted by lens 2 (a green one) an cross at the tip of arrow RI thus forming the real image produced by the two-lens system. Ray a is chosen is such a way so after being refracted by lens 1 it passes through the center of lens 2, so lens two does not change its direction. Ray b passes through the focus of lens 1, so after refraction by lens one it propagates parallel to the optical axis. After refraction by lens 2, ray b passes through the focus of lens 2.

For lens 1 we can write:

$$\frac{1}{s_o^1} + \frac{1}{s_i^1} = \frac{1}{f_1} \qquad (1)$$

For lens 2 we have:

$$\frac{1}{s_o^2}+\frac{1}{s_i^2}=\frac{1}{f_2} \qquad (2)$$

But s_o^2 is the distance to a virtual object, so it has to be negative: $S_o^2 = -(s_i^1 - t) \approx -s_i^1$, since t is much smaller than any distance in the system. So equation (2) becomes:

$$-\frac{1}{s_i^1}+\frac{1}{s_i^2}=\frac{1}{f_2} \qquad (3),$$

And being combined with equation (1) gives:

$$\frac{1}{s_o^1}+\frac{1}{s_i^2}=\frac{1}{f_2}+\frac{1}{f_2} \qquad (4)$$

So, introducing the combined focal distance as:

$$\frac{1}{F}\equiv\frac{1}{f_2}+\frac{1}{f_2} \qquad (5)$$

We have for a two-lens system:

$$\frac{1}{S_O}+\frac{1}{S_i}=\frac{1}{F} \qquad (6)$$

Equation (6) can be rewritten as:

$$V_O + V_i = D \qquad (7)$$

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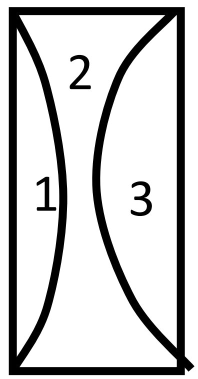
Here we denoted ***1/s_o*** as $\boldsymbol{V_o}$. Parameter $\boldsymbol{V_o}$ is called ***vergence*** since it is related to the curvature of the wavefront propagating from the point object. Parameter ***D=1/F*** is called optical or ***refractive power*** of the lens. When the distances in optics are measured in meters, their reciprocals are the units called "***diopters***". Thus the refractive power of a lens with a focal length of 20 cm is 5 diopters (5 dpt). We just learned that net optical power of a two thin lens system, where the lenses are placed together is equal to the sum of the optical power of the lenses. This rule work for more than 2 lenses:

$$D_1 + D_2 + \cdots + D_n = D \qquad (8)$$

The total width of the lens stack (lens thicknesses plus separation distances) has to be much less than the focal distance of the system (1/D).

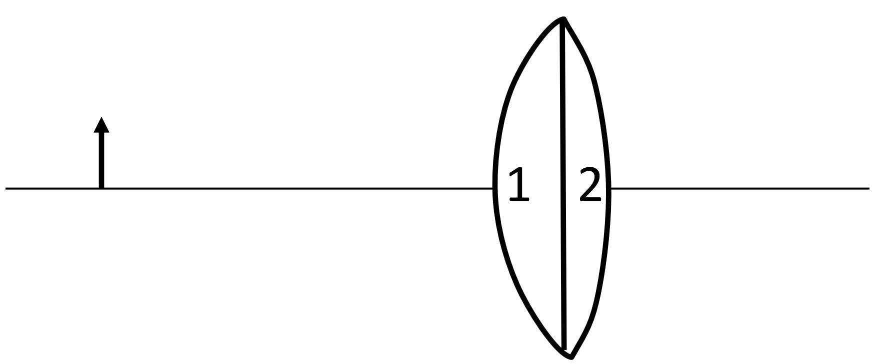
Problems:

1. There are 3 lenses made from a glass plate. (see picture below).



The system consisting just from the lenses 1 and 2 has the refractive power of -2dpt, the refractive power of the lenses2 and 3 together is -3dpt. Find the refractive power of the lens

2. There is a system consisting of two lenses and an object (See Figure below). The distance between the object and the lenses is higher than the focal distances of each lens. If we will remove the lens 2, the magnification is 2; if we will remove the lens 1, then the magnification is 3. What is the magnification if we keep both lenses?



3. The distance between a candle and a screen is 20cm. If you put a convex lens with a focal distance of 4/5 cm in between the object and the screen, a sharp image of the candle will be projected on the screen. There are two such positions for the lens. Find

them. (Hint: take the distance from the object to the lens as **x**, and the distance from the lens to the screen as **20-x**. Then try to use our “main lens equation”)