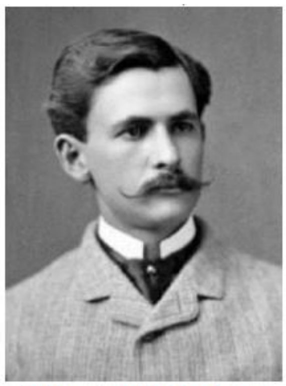
Homework 12.

Speed of light. Michelson-Morley experiment.

As we discussed light as waves, there was a question: is there the media which "produces" light waves like water "produces" the waves on the water surface and air "produce" sound waves? In the end of 19^{th} century the physicists believed that such a media exists and called it "luminiferous ether". As long as the Earth moving in space, theoretically we could measure the speed of the Earth with respect to the ether. To check this assumption, Albert Michelson and Edward Morley in 1887 performed their famous experiment in an attempt to register the motion of Earth through the ether by optical methods.



Albert Michelson (1852-1931).

Edward Morley (1838-1923).

As it was believed, the ether is a substance which penetrates the universe and is the medium for the propagation of light like, say, air is the medium for the sound waves. The motionless ether provides a good reference frame to register the motion of Earth. Assuming the velocity addition rule from classical mechanics the scientists expected to detect the difference in the speed of light propagating along and perpendicular to the direction of the Earth motion. A.A. Michelson invented the optical interferometer – very sensitive device which made such detection possible.

In the description of the experiment below I follow the book (a really nice book!) of R. Resnick and D. Halliday "Basic concepts in relativity and early quantum theory".

The device is shown schematically in Figure 2a. The beam of light from the light source is split by the partially silvered mirror (we will call it "beamsplitter") into two beams which are shown in red and blue in Figure 2a. "Red" and "blue" beams are coherent since they originate from the same source. After reflection from the mirrors, the beams "meet" at the telescope objective and produce interference pattern: the alternating bright and dark stripes (Insert in Figure 2a)). The interferometer was mounted on a massive stone slab for stability. The slab was floated in mercury, so it can be smoothly rotated around the central pin.

The interference pattern, i.e. the positions of the dark and bright stripes, depend on the phase difference between two beams. Let us compute this difference. First, let is assume that the Earth velocity **v** (green arrow in Figure 2a) is directed along the blue beam (beam 1). This difference occurs because of two causes: different lengths of the path traveled and different speeds of travel with respect to the instrument due to "ether wind". The time for beam 1 to travel from beamsplitter M to mirror 1 and back is:

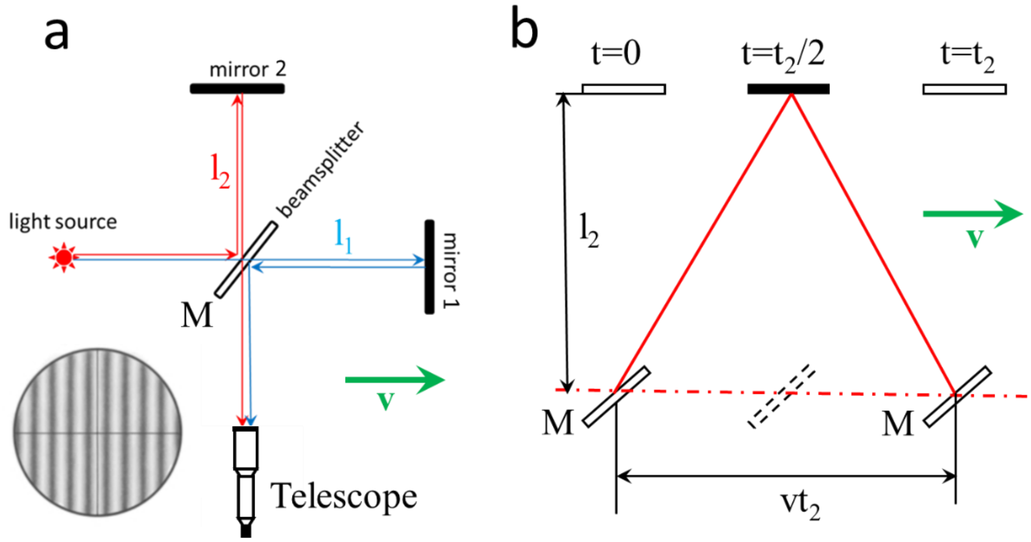


Figure 2. Simplified version of Michelson interferometer (a). Insert: the interference pattern, seen in the telescope. Cross-stream path of the "red" beam in the assumption that the device is moving with respect to ether.

$$t_1 = \frac{l_1}{c-v} + \frac{l_2}{c+v} = l_1\left(\frac{2c}{c^2-v^2}\right) = \frac{2l_1}{c}\left(\frac{1}{1-v^2/c^2}\right) \quad (1),$$

here $\boldsymbol{c}$ is the speed of light moving through the ether.

The light traveling from M to mirror 1 has an "upstream velocity of $\boldsymbol{c-v}$ with respect to the apparatus, and the reflected light has "downstream" velocity $\boldsymbol{c+v}$.

The path of the "red" beam (beam 2) is a cross-stream path through the ether (Figure 2b). The travel time t_2 for this beam can be calculated using the expression:

$$2\sqrt{l_2^2 + \left(\frac{vt_2}{2}\right)^2} = ct_2 \quad (2)$$

The left part of the equation is the beam path length obtained using Pythagorean theorem, the right part is the same length expressed through the speed of light and the travel time. Resolving (2) with respect to the time we obtain:

$$t_2 = \frac{2l_2}{\sqrt{c^2-v^2}} = \frac{2l_2}{c}\frac{1}{\sqrt{1-v^2/c^2}} \quad (3)$$

It should be noted that we calculated time t_1 in the reference frame of the apparatus, while t_2 was calculated in the reference frame of the ether. But, according to the classical mechanics, the time is absolute and flows the same way in both the reference frames. In both cases the travel time is increased. The difference Δt between the travel times is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{l_2}{\sqrt{1-\beta^2}} - \frac{l_1}{1-\beta^2}\right), \quad \beta \equiv \frac{v}{c} \quad (4)$$

Here we introduced the parameter β, called "the speed parameter"

If we will rotate the interferometer through 90°, $\boldsymbol{l_1}$ will become the cross-stream length and $\boldsymbol{l_2}$ will become the downstream one. We will designate the corresponding times by primes and the travel time difference will be:

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{l_2}{1-\beta^2} - \frac{l_1}{\sqrt{1-\beta^2}}\right), \qquad (5)$$

Hence, the rotation of the interferometer changes the time difference by:

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_2+l_1}{1-\beta^2} - \frac{l_2+l_1}{\sqrt{1-\beta^2}}\right), \qquad (6)$$

Taking into account that $\boldsymbol{\beta}$ is very small (~ 10^{-4}), we can use approximate expressions:

$$\frac{1}{1-\beta^2} \approx 1 + \beta^2 \; ; \; \frac{1}{\sqrt{1-\beta^2}} \approx 1 + \frac{1}{2}\beta^2 \qquad (7)$$

Then, expression (6) can be simplified as:

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left(1 + \beta^2 - 1 - \frac{1}{2}\beta^2\right) = \left(\frac{l_1+l_2}{c}\right)\beta^2, \qquad (8)$$

Therefore, the rotation of the instrument should cause the change in the travel time difference along the paths and, hence, the shift in the interference pattern.

No shift was detected. The experiment was later repeated many times at higher accuracy level – the result was negative. The speed of light was measured the same –independent from the direction of light propagation with respect to the Earth motion. All experimental attempts to detect the "ether wind" (in other words "to measure absolute motion") failed.

Henri Poincaré, famous French mathematician and physicist suggested that the experiments failed not because of a lack of the experimental skill, but because it is a fundamental physical low: "detection of absolute velocity is impossible".

Henri Poincaré

(1854-1912).

We know very well that all physics laws are invariant in all inertial reference frames (for example, Newton's second law). This is the Galileo's principle of relativity. The word *relativity* here means that the statement "the object is moving at a constant velocity" makes sense only if we specify the reference frame in which we are monitoring the position of the object. Just to remind:

a reference frame is an object or a group of objects which we use to specify the spatial position of a physical body. If we know one inertial reference frame, all the reverence frames moving at constant velocities with respect to the initial reference frame are also inertial.

Imagine that we specify the position of an object A in space with respect to the certain reference frame XYZ. In our 3-dimensional world we have to specify 3 numbers: x, y and z coordinates.

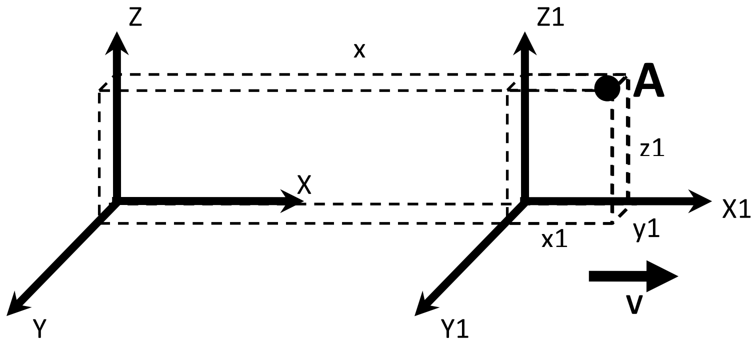


Fig.1

Imagine that we have another reference frame$X_1Y_1Z_1$ which moves at a velocity ***V*** with respect to the XYZ frame along the x axis (see Fig.1). The object A which is at rest in XYZ is moving in $X_1Y_1Z_1$. Assume that initially these reference frames had same origin. Now we can express the coordinates of **A** in the "moving" reference frame x_1, y_1, z_1 through the x, y and z in the following way:

$$\begin{aligned} x_1 &= x - Vt \\ y_1 &= y \\ z_1 &= z \end{aligned} \qquad (1)$$

Here *t* is time. If A is moving in the XYZ at a velocity ***v*** along the X axis we can use simple rule of velocity addition:

$$\boldsymbol{v}=\boldsymbol{v_1}+\boldsymbol{V} \qquad (2)$$

Here $\boldsymbol{v_1}$ is the velocity of A in the $X_1Y_1Z_1$.

But, as the results of the Michelson-Morley experiment indicate, the speed of light in vacuum is the same independently of the velocity of the reference frame. The motion faster than the light in vacuum is impossible. This absolutely contradicts to the velocity addition rule which is experimentally checked so many times!

Next time we discuss what would be the solution of this problem.

Problem:

Michelson and Morley were able to obtain an optical path length (l_1+l_2) of about 22m. Taking the light wavelength as λ=5.5x10^{-7}m, estimate the ratio between $\Delta t' - \Delta t$ and the period of the light wave. This ratio indicates the expected shift in the observed interference pattern.