Homework 17

<u>Relativistic velocity composition rule.</u>

We are going back to then old question. “If the headlight of a moving train is turned on what will be the velocity of the light, propagating from the train, with respect to an observer on the ground?” According to the classical velocity composition rule this is the sum of the train’s velocity with respect to the ground and the light velocity with respect to the train. We know now that this is not correct. We need another velocity composition rule which would be in agreement with the results of Michelson-Morley experiment, i.e. with the statement that the speed of light does not depend on the choice of inertial reference frame.

Again, we have object A in space with respect to the certain reference frame XYZ. But now this object is moving parallel to X axis and opposite to the axis direction:

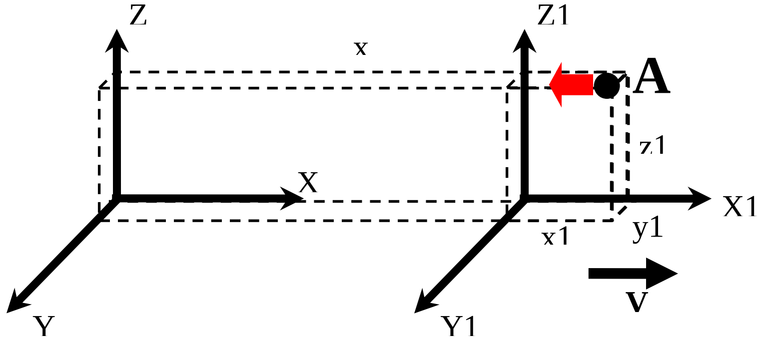

Fig.1

Let us denote the velocity of point A in the XYZ frame as ***v***. Imagine that we have another reference frame$X_1Y_1Z_1$ which moves at a velocity ***V*** with respect to the XYZ frame along the x axis (see Fig.1). Using the Lorentz transformations we can write the coordinate ***x’*** and time ***t’*** of point A in the moving reference frame:

$$x' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$t' = \frac{t - \frac{Vx}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (3)$$

Let point A moves from position x'_1 to position x'_2 in the moving reference of frame. Then:

$$\Delta x' = x_2' - x_1' = \frac{\Delta x + V\Delta t}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (4)$$

$$\Delta t' = t_2' - t_1' = \frac{\Delta t + \frac{V\Delta x}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (5)$$

The velocity v’ of point A in the moving reference of frame is:

$$v' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x + V\Delta t}{\Delta t + \frac{V\Delta x}{c^2}} \qquad (6)$$

In the last equality let us divide the numerator and denominator by Δt and take into account that $\frac{\Delta x}{\Delta t} = v$, which is the velocity of point A with respect to the ground (system XYZ).

$$v' = \frac{v+V}{1+\frac{Vv}{c^2}} \qquad (7)$$

Expression (7) is the relativistic velocity composition rule. It is seen from (7) that even two objects are moving to each other at the velocities approaching to ***c*** with respect to the ground, the velocity of one with respect to the other is still less than ***c***.

Problems:

1. Two rockets are moving toward each other at a velocities 0.7c each (these velocities are measured with respect to the Earth). What is the velocity of one rocket with respect to the other?
2. *An astronaut is moving at a speed of 5km/hour inside a spacecraft in the direction perpendicular to the spacecraft’s motion. How an observer on Earth is “seeing” this component of the astronaut’s velocity if the velocity of the spacecraft is $1.8 \cdot 10^8 m/s$? (To solve this problem you have take a look at the Lorentz transformations again).
3. Using relativistic velocity composition rule, show that the speed of light is the same in both XYZ and X’Y’Z’.