<u>Introduction to mechanics</u>

The key to solving a physical problem is usually in its question. It is a good strategy to think about the question first. In some problems of the homework the question is

What is average speed?

To start solving the problem one has to find out what does “average speed” means? Average speed is a rate of total distance and time interval required to cover this distance.

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

For example, you have to go for 1km. First you run, then stop for a while to take a break and finally you walk. It took 15 minutes to cover 1 km. The average speed in this case is

$$\text{average speed} = \frac{\text{total distance} = 1\text{km} = 1000\text{m}}{\text{total time} = 15\text{min} = 15 \text{ x } 60\text{s} = 900\text{s}} \approx 1.11\frac{m}{s}$$

It means that instead of running, taking a rest and, finally, walking you just keep going with a uniform speed of 1,11m/s you will pass 1 km for the same time of 15min.

After you found out what is average speed, it’s time to take a look again at the text of the problem and think how you can calculate total distance and total time using the data of the problem.

Another question is

What is average velocity?

Average velocity is a rate of total ***displacement*** and time interval required to cover this distance.

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} \quad (1)$$

For example, if at the end of a very long trip you returned to the starting point, your average velocity is zero, because your displacement is zero.

Average speed and average velocity have same magnitude in case you move along a strait line in one direction.

Velocity composition rule

1.The velocity can only be measured with respect to an object. Any time we say that the velocity is, say, 5miles per hour we have to specify with respect to what object this velocity is measured. We will call this object as " the frame of reference". When we say that the velocity of the car is 50 km/h it usually means that this is the velocity with respect to the ground. A physicist would say that the velocity is 50km/h in the ground's reference frame. The velocity (the speed and the direction of motion) depends on the choice of the reference frame.

2.The velocity addition rule is given below:

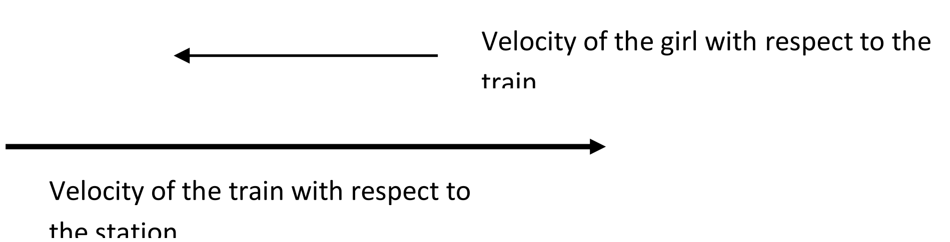
If the object B moves with respect to the object A at the velocity V1 and the object C moves with respect to the object B at the velocity V2, then the object C moves with respect to the object A at the velocity V=V1+V2.

1. Please read the example below - I hope it will be helpful.

Problem *A girl is walking in the moving train in the direction opposite to the train's motion. You are watching the passing train staying at the station. The velocity of the girl with respect to the train is 1m/s, the velocity of the train with respect to the ground is 36 km/h. Find the velocity of the girl with respect to you.*

Solution.

1. *Make a picture*



2. *Choose the" positive" direction. You have two options:" left to right" or" right to left". You can pick up any one – the result will not depend on your choice. For this problem I choose left to right. From now on, all the velocities directed left to right are positive, all the velocities "looking" in the opposite directions are negative. The velocity of the train is positive, the velocity of the girl with respect to the train is negative, because it "looks" in the opposite direction.*
3. *According to the velocity composition rule:*

Velocity of the girl with respect to the station (we denote it as $\boldsymbol{V_{gs}}$) **=** *velocity of the girl with respect to the train ($\boldsymbol{V_{gt}}$)* **plus** *velocity of the train with respect to the station ($\boldsymbol{V_{ts}}$).Or*

$$\boldsymbol{V_{gs}=V_{gt}+V_{ts}}.$$

4. *According to the problem,* $\boldsymbol{V_{gt}}$= - 1m/s (it is negative), $\boldsymbol{V_{ts}}$= 36km/h=10m/s (it is positive). So,

$$\boldsymbol{V_{gs}} = -1\text{m/s} + 10\text{m/s} = 9\text{m/s}$$

The problem is solved. The answer is positive 9m/s. It means that the girl is moving left to right (since the result is positive) at a speed of 9m/s.

Average velocity

When the speed of the moving object (a car, a plane etc.) changes as the object moves it is not possible to characterize this motion by a certain speed. But it is possible to introduce *average speed*. To find average speed we have to take the distance passed by, say, a car and divide it for the time which was spent to pass the distance. For a first glance there is nothing new: we always calculate the speed this way. Looking more attentively we understand that there is an important difference: average speed is not the *actual* speed of the car. The meaning of average speed is the following: if the car would have moved at the constant speed which is equal to the average speed, it passed the same distance for the same time.

Unlike the average speed, average velocity is vector, so it has both magnitude and direction. To find average velocity we have to take the displacement between the starting and final points and divide it to the total time which is required to complete the motion.

Acceleration

In everyday life we use the word *acceleration* to describe increase of the speed of a moving object. Acceleration in physics has a different meaning. It is change in *velocity* per unit time. Any time the speed and/or the direction of motion of an object changes we deal with *accelerated* motion. An example of acceleration motion is falling. We know that any object falls down with acceleration of ~10m/s^2 (9.8 m/s^2, to be exact).

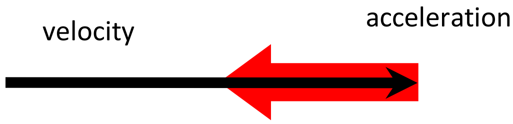
Acceleration is a vector – it has both magnitude and direction.

For the case of rectilinear motion (just to remind – this is the motion along a straight line) there are two major cases:

1. Acceleration is directed along the velocity.

In this case the velocity and acceleration have same sign and speed of the object is *increasing* with time. The acceleration magnitude gives us the rate of the speed increase. For example acceleration of 5meters per second per second (this is not a typo!) means that the speed increases for 5m/s every second. It is usually denoted as $5m/s^2$ (five meter per second square)

2. Acceleration is directed oppositely to the velocity.



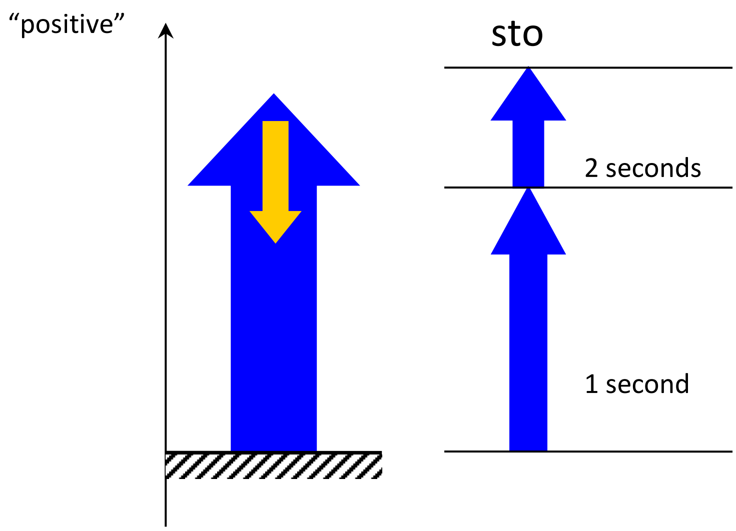
In this case the velocity and acceleration have opposite signs and speed of the object is *decreasing* with time. The acceleration magnitude gives us the rate of the speed decrease. For example, acceleration of -5meters per second per second means that the speed decreases for 5m/s every second.

For some complicated types of motion (oscillations of a pendulum, for example) acceleration changes with time. We will study only the motion at a constant acceleration. If we know acceleration and initial speed we can easily find the speed at any later moment:

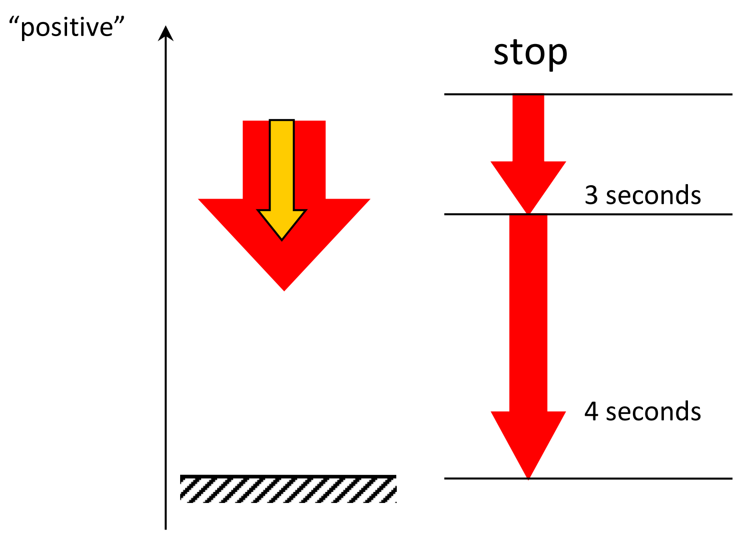
$$\vec{V} = \vec{V}_0 + \vec{a} \cdot t$$

Velocity after the time *t* = Initial velocity plus Acceleration multiplied by time

We discuss now one simple example. This examples show how does *acceleration* work. Imagine that you throw a pebble vertically up (if you decide to make this experiment please do not forget to change your location immediately after the pebble's start) at the velocity of 20m/s. What will happen to the pebble after?



As usual, first let us choose “positive” direction. We know that after we let the pebble go it experiences acceleration due to gravity. This acceleration is directed down and equals $10m/s^2$. The initial velocity of the pebble is directed up. This is along our “positive” axis so the initial velocity is positive. The acceleration due to gravity is always directed down (to be exact it is directed to the center of Earth). This is opposite to our “positive” direction, so the acceleration is negative and directed opposite to the velocity. This means that the initial speed decreases for 10m/s every second. In one second after the start the velocity is 10m/s, in two seconds the velocity is zero -the pebble stops. But the acceleration is still there and continues subtracting 10m/s from the pebble’s velocity every second. In three seconds the velocity is -10m/s.



It is negative now. It means that it is now directed against our positive axes and “looks” down. Now both the velocity and acceleration “look” the same side. As we know, in this case the acceleration increases the speed. In 4 seconds the speed is again 20m/s but the pebble moves down, so the velocity is -20m/s.

In spite of the velocity is changing along the pebble’s path, the acceleration is the same in each point of the path and equals -10m/s^2.

Distance passed by uniformly accelerated object

Uniform acceleration (constant acceleration) means that the acceleration does not change as the object is moving.

Example

A car spent time *t* moving with positive acceleration *a* from point A to point B along a straight line. A speed of the car at the point A was $V_{initial}$. We know that the motion of the car is accelerated and it moves along a straight line. It means that the *speed* of the car is increasing every moment . In time *t* after the car started from the point A its speed is

$$V_{final} = V_{initial} + at \quad (1)$$

In the case of <u>uniform acceleration</u> the average speed can be calculated as:

$$V_{average} = \frac{V_{initial} + V_{final}}{2}$$

Now, to calculate the distance *S* we have just to multiply the average speed by the time:

$$S = V_{average} \cdot t$$

$$V_{average} = \frac{V_{initial} + V_{final}}{2} = \frac{V_{initial} + V_{initial} + at}{2} = \frac{2 \cdot V_{initial} + at}{2} = V_{initial} + \frac{at}{2}$$

Now, to calculate the distance *S* we have just to multiply the average speed by the time:

$$S = V_{average} \cdot t = \left(V_{initial} + \frac{at}{2} \right) \cdot t = V_{initial} \cdot t + \frac{a \cdot t \cdot t}{2} = V_{initial} \cdot t + \frac{a \cdot t^2}{2}$$

The signs before $V_{initial}$ and *a* we choose according the direction. For a negative acceleration (if the car stops) we have:

$$S = V_{initial} \cdot t - \frac{a \cdot t^2}{2}$$

Force and First Newton's law

Any physical body being in motion tends to stay in motion (if the body is at rest it tends to stay at rest). This sentence is called "the law of inertia". It means the following: *velocity* of an object does not change unless the object will interact somehow with other objects. Nothing starts moving by itself and stops by itself. We need another object to change velocity, or, using another words, to create acceleration. The interaction which causes acceleration of an object is called *force*. Force is a vector: it has both magnitude and direction. Examples of the force are: gravity force, electric and magnetic forces, elastic force, friction force.

Let us consider the following imaginary experiment: if you push with the same way an empty shopping cart and a heavily loaded shopping cart. The first one will move faster (check it). A physical quantity which expresses the property of an object to resist acceleration is called *mass*. The mass is measured in kilograms (kg) and grams (g). 1kg=1000g

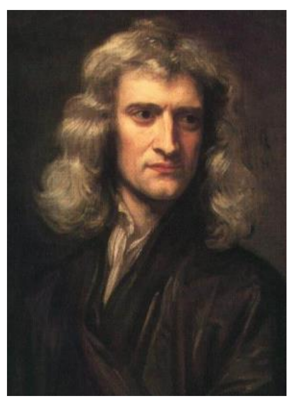
It is very important not to mix mass and weight. The weight (in common, “everyday” meaning of this word) depends on how strong an object presses to the surface supporting the object. Weight of the same object is different on different planets. The mass express the fundamental property of an object to resist acceleration. Any object with nonzero mass will resist acceleration even in deep space.

Second Newton’s law

The force can be determined as interaction which makes the interacting object accelerate. Force and acceleration are connected by a simple formula:

$$\vec{F} = m\vec{a}$$

Here *F* is a force applied to an object, *m* is the mass of the object and *a* is the acceleration of the object. Force is measured in newtons (N). 1N is the force required to provide an acceleration of 1m/s^2 to an object with a mass of 1kg.The unit of force is named after Sir Isaac Newton (1643-1727)– one of the brightest genii in human history.



Sir Isaac Newton (www.wikipedia.org)

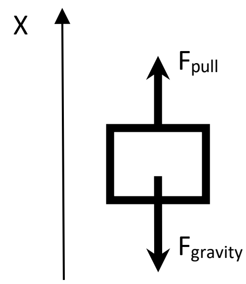
Arrows over “*F”* and “*a”* remind us that both force and acceleration are vector quantities, which means that they have both magnitude and direction. You can see from the formula that the more mass of the object the more force is needed to provide same acceleration. A heavy object is difficult to accelerate.

However, if an object is not accelerating it does not mean that no forces applied to the object. In most of the cases it just means that forces applied to the object compensate each other. In other words, the sum of all forces applied to the object is zero. So the **force *F* in the formula**

above is the sum of all the forces applied to the object. We will call this sum as *total net force*. How we can sum forces?

Example: You pull up a 10kg load with a force of 150N. Is this force enough to lift the load? What is acceleration of the load?

Solution: First, let us make a picture



Let us choose "positive" direction as "down to up". So the "pulling" force is positive because it looks up and the gravity force is negative because it looks down:

$$F_{pull} - F_{gravity} = ma$$

or

$$F_{pull} - mg = ma$$

We do not know yet what the acceleration (magnitude and sign) is. Let us calculate it:

$$150N - 10kg \cdot 9.8\frac{m}{s^2} = 10kg \cdot a$$

$$a = \left(150N - 10kg \cdot 9.8\frac{m}{s^2}\right) \div 10kg = 5.2\frac{m}{s^2}$$

The acceleration is positive. It means that it is directed up, along our "positive" axis. It also means that the applied force is enough to lift the load.

Third Newton's law

The first Newton's law can be simply formulated as follows: every time one object applies force to another object, this "another" object applies force to the first object. The forces have equal magnitudes and opposite directions.

For example, imagine that you try pushing a heavy 20 kg stone while both you and the stone are on ice. You push the stone with a force of 80N. What happens next?

Both you and the stone will slide in opposite directions. What is acceleration of the stone while you are pushing it? It is simple to calculate it:

$$a_{stone} = \frac{F}{m} = \frac{80N}{20kg} = 4\frac{m}{s^2}$$

What about your acceleration? Assume that your mass is 40kg. But what about force which made you slide? Its magnitude is equal to the magnitude of the force you applied to the stone, but it is directed oppositely. It looks like the stone pushed you with the same force of 80N:

$$a_{you} = \frac{F}{m} = \frac{80N}{40kg} = 2\frac{m}{s^2}$$

So your acceleration is be smaller, because your mass is higher and the magnitude of the forces applied to you and the stone are same.

This "picture" is universal. Whenever you apply force to something this something applies force of equal magnitude and opposite direction to you. *These forces do not compensate each other because they are applied to different objects.* We know that we can add and subtract only the forces applied to the same object.

Now we learned three laws which are the base of simple mechanics:

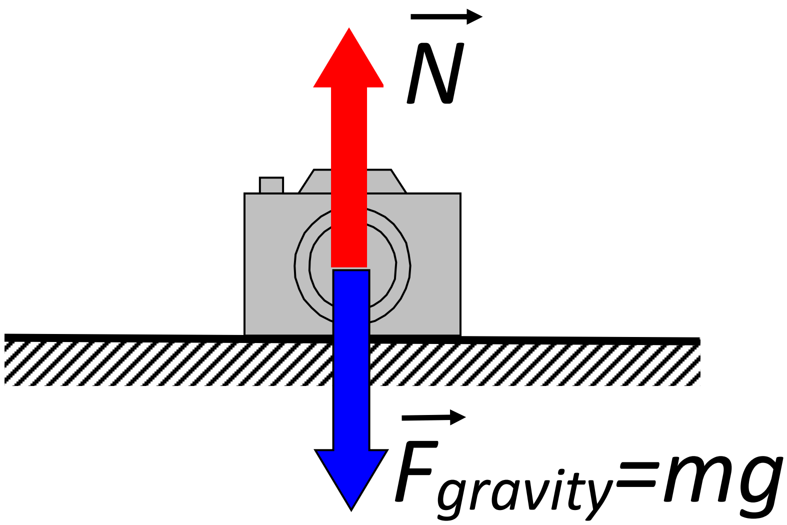
1. The object in motion tends to stay in motion; an object at rest tends to stay at rest.
2. The total net force applied to an object is equal to the mass of the object multiplied by the acceleration of the object.
3. Any time a force is applied by one object to another, a force of same in magnitude and opposite direction is applied to the first object by the second one.

These laws are called Newton's laws of motion.

Normal force and friction force

Normal force.

Any time we put an object (say, a pen) on a table, floor or any other surface this object apply force to this surface. The origin of this force may be just the gravity (the pen is attracted by Earth). We can also apply additional pressure to the pen. We observe that the pen does not move in vertical direction – it just lies on the table. This means that in spite of the gravity force applied to the pen, the acceleration of the pen in vertical direction is zero. This, in turn, means that the gravity force is compensated by some other force or forces. According to the third Newton's law the surface applies the force of equal magnitude and opposite direction to the object. This force does not allow the pen to go down through the table. We will call this force as "normal force". Normal force is directed perpendicularly to the surface. (Just to remind: two straight lines are called perpendicular if they cross at the right angle. A straight line is called perpendicular ("normal") to the plane if the line is perpendicular to any straight line belonging to the plane)



As we can see in the picture, if the camera just lays on the table, the magnitude of the normal force is equal to the magnitude of the gravity force.

$$ma = N - mg = 0$$

$$N = mg$$

Here our "positive" axis is directed up.

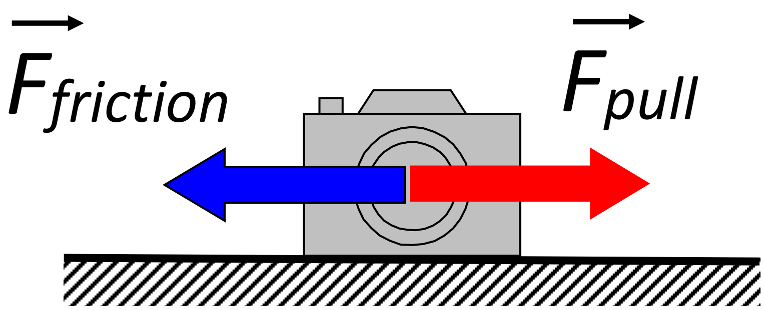
If we will press the camera down with a force F_{press} the normal force will increase to compensate both the gravity force and the pressure force.

$$ma = N - mg - F_{press} = 0$$

$$N = mg + F_{press}$$

a) ***Friction force*** *(we will discuss it in detail during next class)*

When we try to push or pull a heavy box standing on the floor it may not move in spite of a considerable pulling of pushing force applied. Some force (or forces) applied to the box by the surface compensates the pushing force and the acceleration in the "pushing" direction is zero. If the magnitude of pushing (or pulling) force is less than certain magnitude which we will call *static friction force*, the box will not move and friction force magnitude is equal to this of the pushing force. If we increase the pushing force, the friction force increases as well until the static friction force is reached. After that, the friction force does not increase anymore and, if we increase the pushing force just a little bit, the box will start moving.



$$ma = F_{friction} - F_{pull} = 0$$

$$F_{friction} = F_{pull}$$

How to calculate the static friction force F_{fs}? The magnitude of the static friction force is proportional to the magnitude of the normal force. Speaking "common sense language" the heavier the box the stronger we have to push to move it.

$$F_{fs} = \mu \cdot N$$

Here μ is the coefficient of friction. This is a number which depends of the object (box) and surface materials and the roughness of the surfaces. If the surfaces are rough, this number is large, so more force is required to move the object.

After the box started moving the friction force is equal to μN. Strictly speaking this is not always correct and, in some cases, the friction force applied to a moving object (dynamic friction force) is not equal to the static friction force. This time we will not discuss this effect in details and, for simplicity, assume that the static friction force is equal to the dynamic one.