The key to solving a physical problem is usually in its question. It is a good strategy to think about the question first. In some problems of the homework the question is

What is average speed?

To start solving the problem one has to find out what does “average speed” means? Average speed is a rate of total distance and time interval required to cover this distance.

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

For example, you have to go for 1km. First you run, then stop for a while to take a break and finally you walk. It took 15 minutes to cover 1 km. The average speed in this case is

$$\text{average speed} = \frac{\text{total distance} = 1\text{km} = 1000\text{m}}{\text{total time} = 15\text{min} = 15 \text{ x } 60\text{s} = 900\text{s}} \approx 1.11\frac{m}{s}$$

It means that instead of running, taking a rest and, finally, walking you just keep going with a uniform speed of 1,11m/s you will pass 1 km for the same time of 15min.

After you found out what is average speed, it’s time to take a look again at the text of the problem and think how you can calculate total distance and total time using the data of the problem.

Another question is

What is average velocity?

Average velocity is a rate of total ***displacement*** and time interval required to cover this distance.

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} \quad (1)$$

For example, if at the end of a very long trip you returned to the starting point, your average velocity is zero, because your displacement is zero.

Average speed and average velocity have same magnitude in case you move along a strait line in one direction.

Velocity composition rule

Last class we discussed the velocity addition rule. What are the most important things we have learned?

1.The velocity can only be measured with respect to an object. Any time we say that the velocity is, say, 5miles per hour we have to specify with respect to what object this velocity is measured. We will call this object as " the frame of reference". When we say that the velocity of the car is 50 km/h it usually means that this is the velocity with respect to the ground. A physicist would say that the velocity is 50km/h in the ground's reference frame. The velocity (the speed and the direction of motion) depends on the choice of the reference frame.

2.The velocity addition rule is given below:

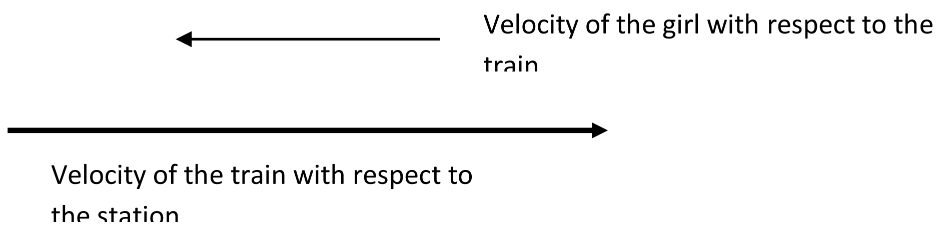
If the object B moves with respect to the object A at the velocity V1 and the object C moves with respect to the object B at the velocity V2, then the object C moves with respect to the object A at the velocity V=V1+V2.

1. Please read the example below - I hope it will be helpful.

Problem *A girl is walking in the moving train in the direction opposite to the train's motion. You are watching the passing train staying at the station. The velocity of the girl with respect to the train is 1m/s, the velocity of the train with respect to the ground is 36 km/h. Find the velocity of the girl with respect to you.*

Solution.

1. *Make a scheme*



2. *Choose the" positive" direction. You have two options:" left to right" or" right to left". You can pick up any one – the result will not depend on your choice. For this problem I choose left to right. From now on, all the velocities directed left to right are positive, all the velocities "looking" in the opposite directions are negative. The velocity of the train is positive, the velocity of the girl with respect to the train is negative, because it "looks" in the opposite direction.*
3. *According to the velocity composition rule:*

Velocity of the girl with respect to the station (we denote it as V_{gs}*) = velocity of the girl with respect to the train (*V_{gt}*)* **plus** *velocity of the train with respect to the station (*V_{ts}*).Or*

$$V_{gs}=V_{gt}+V_{ts}.$$

4. *According to the problem,* V_{gt}= - 1m/s (it is negative), V_{ts}= 36km/h=10m/s (it is positive). So,

$$V_{gs}= -1\text{m/s} +10\text{m/s} = 9\text{m/s}$$

The problem is solved. The answer is positive 9m/s. It means that the girl is moving left to right (since the result is positive) at a speed of 9m/s.

Now the homework:

1. Choose the direction right to left as the "positive" and solve the problem given above. Is the result changed?
2. A boat moves up the river (against the water flow).The speed of the boat with respect to the water is 5m/s, the speed of the water with respect to the river bancs is 1m/s. Find the velocity of the boat with respect to the river bancs.
3. Two cars a moving along a straight road toward each other from different towns. The speed of the right car is 72km/h, the speed of the left car is 36km/h. Find the velocity of the right car with respect to the left car.

Average velocity

Last class we discussed average speed and average velocity. When the speed of the moving object (a car, a plane etc.) changes as the object moves it is not possible to characterize this motion by a certain speed. But it is possible to introduce *average speed*. To find average speed we have to take the distance passed by, say, a car and divide it for the time which was spent to pass the distance. For a first glance there is nothing new: we always calculate the speed this way. Looking more attentively we understand that there is an important difference: average speed is not the *actual* speed of the car. The meaning of average speed is the following: if the car would have moved at the constant speed which is equal to the average speed, it passed the same distance for the same time.

Unlike the average speed, average velocity is vector, so it has both magnitude and direction. To find average velocity we have to take the displacement between the starting and final points and divide it to the total time which is required to complete the motion.

1. A car passed 30km at the speed of 15m/s. Then the car spent 1 hour to pass 40 km. What was the average speed of the car?
2. It took 2.5 min for the train to pass the bridge at the speed of 18km/h. The length of the bridge is 630m. What is the length of the train?
3. A motorcycle moves at the speed 5m/s for 15s, then it moves at the speed of 8m/s for 10s and, finally, it moves at the speed of 20m/s for 6c. What is the average velocity of the motorcycle?
4. A walker passed one halve of the distance at the speed of 1m/s, the other half was passed at the speed of ½ m/s. What was the average speed of the walker? (*This problem is a bit more challenging. To solve it try to answer to the following question: do we need to know the distance to solve the problem? Does the average speed in this problem depend on the total distance passed by the walker?)*
5. Venice is connected to the continental part of Italy by the bridge whose length is 4km 70m. A bicyclist passes the bridge for 6min 47 s. The car starts moving in the same direction later than the bicyclist but they "finish" together (at the same time) at the end of the bridge. The speed of the car is 4m/s higher than the speed of the bicyclist. Find the "delay" time for the car.

Acceleration

Last time we discussed *acceleration*. In everyday life we use the word *acceleration* to describe increase of the speed of a moving object. Acceleration in physics has a different meaning. It is change in *velocity* per unit time. Any time the speed and/or the direction of motion of an object changes we deal with *accelerated* motion. An example of acceleration motion is falling. We know that any object falls down with acceleration of ~10m/s^2 (9.8 m/s^2, to be exact).

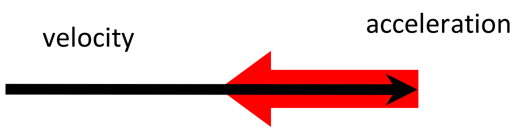
Acceleration is a vector – it has both magnitude and direction.

For the case of rectilinear motion (just to remind – this is the motion along a straight line) there are two major cases:

1. Acceleration is directed along the velocity.

In this case the velocity and acceleration have same sign and speed of the object is *increasing* with time. The acceleration magnitude gives us the rate of the speed increase. For example acceleration of 5meters per second per second (this is not a typo!) means that the speed increases for 5m/s every second. It is usually denoted as $5m/s^2$ (five meter per second square)

2. Acceleration is directed oppositely to the velocity.



In this case the velocity and acceleration have opposite signs and speed of the object is *decreasing* with time. The acceleration magnitude gives us the rate of the speed decrease. For example, acceleration of -5meters per second per second means that the speed decreases for 5m/s every second.

For some complicated types of motion (oscillations of a pendulum, for example) acceleration changes with time. We will study only the motion at a constant acceleration. If we know acceleration and initial speed we can easily find the speed at any later moment:

$$\vec{V} = \vec{V}_0 + \vec{a} \cdot t$$

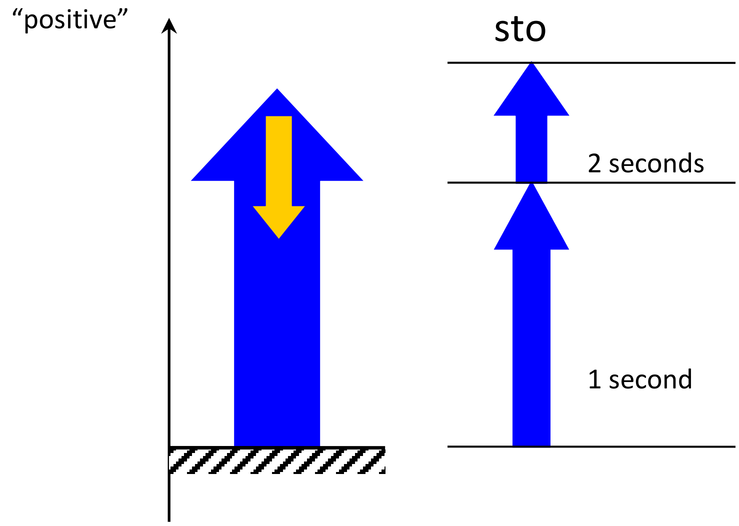
Velocity after the time *t* = Initial velocity plus Acceleration multiplied by time

Problems to solve:

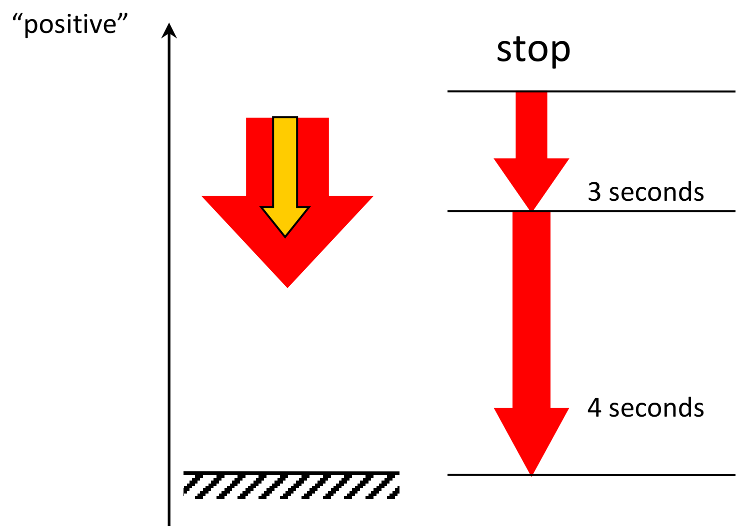
1. Imagine that you dropped a penny from Empire State Building (please, never do it in a real life!). Calculate the speed of the coin in 5 seconds.
2. Explain (and make a scheme) what happens to a pebble if you throw it vertically up?
3. A solder shoots vertically. The bullet starts moving up at a speed of 400m/s. In what time the bullet will stop?

4. A ball falls down from the height of 10m and hits the ground in 1 second. Find average velocity of the ball and compare it to the velocities of the ball in the beginning and in the end of the motion.

We discuss now one simple example. This examples show how does *acceleration* work. Imagine that you throw a pebble vertically up (if you decide to make this experiment please do not forget to change your location immediately after the pebble's start) at the velocity of 20m/s. What will happen to the pebble after?



As usual, first let us choose "positive" direction. We know that after we let the pebble go it experiences acceleration due to gravity. This acceleration is directed down and equals $10m/s^2$. The initial velocity of the pebble is directed up. This is along our "positive" axis so the initial velocity is positive. The acceleration due to gravity is always directed down (to be exact it is directed to the center of Earth). This is opposite to our "positive" direction, so the acceleration is negative and directed opposite to the velocity. This means that the initial speed decreases for 10m/s every second. In one second after the start the velocity is 10m/s, in two seconds the velocity is zero -the pebble stops. But the acceleration is still there and continues subtracting 10m/s from the pebble's velocity every second. In three seconds the velocity is -10m/s.



It is negative now. It means that it is now directed against our positive axes and “looks” down. Now both the velocity and acceleration “look” the same side. As we know, in this case the acceleration increases the speed. In 4 seconds the speed is again 20m/s but the pebble moves down, so the velocity is -20m/s.

In spite of the velocity is changing along the pebble’s path, the acceleration is the same in each point of the path and equals -10m/s²^{2}.

1. I took 10 seconds for a car to increase its speed from 10m/s to 25m/s. The car moves along a straight line. Find acceleration of the car.
2. A tennis ball is sent up at a speed of 40m/s. Find its velocity in 6 seconds after the start. Make a scheme.
3. The ball from the problem 2 reached the maximum height of 160m. In what time the ball reached the highest point of its path? Find the average velocity of the ball at the part of its path between the start and the highest point.

Distance passed by uniformly accelerated object

Last class we have learned how to calculate the distance passed by a uniformly accelerated object. Uniform acceleration (constant acceleration) means that the acceleration does not change as the object is moving.

Example

A car spent time t moving with positive acceleration a from point A to point B along a straight line. A speed of the car at the point A was $V_{initial}$. We know that the motion of the car is accelerated and it moves along a straight line. It means that the *speed* of the car is increasing every moment . In time t after the car started from the point A its speed is

$$V_{final} = V_{initial} + at \quad (1)$$

In the case of <u>uniform acceleration</u> the average speed can be calculated as:

$$V_{average} = \frac{V_{initial} + V_{final}}{2}$$

Now, to calculate the distance S we have just to multiply the average speed by the time:

$$S = V_{average} \cdot t$$

$$V_{average} = \frac{V_{initial} + V_{final}}{2} = \frac{V_{initial} + V_{initial} + at}{2} = \frac{2 \cdot V_{initial} + at}{2} = V_{initial} + \frac{at}{2}$$

Now, to calculate the distance S we have just to multiply the average speed by the time:

$$S = V_{average} \cdot t = \left(V_{initial} + \frac{at}{2} \right) \cdot t = V_{initial} \cdot t + \frac{a \cdot t \cdot t}{2} = V_{initial} \cdot t + \frac{a \cdot t^2}{2}$$

The signs before $V_{initial}$ and a we choose according the direction. For a negative acceleration (if the car stops) we have:

$$S = V_{initial} \cdot t - \frac{a \cdot t^2}{2}$$

Problems:

1. A coin is falling down for 3 sec. An initial velocity of the coin is 0. Find the distance passed by the coin.
2. You send an arrow up at an initial velocity of 20m/s. How high the arrow will go?
3. Let us return to the coin from the problem 1. Find the distance passed by the coin for the third second.

Make detail pictures for all three problems.

Force and First Newton's law

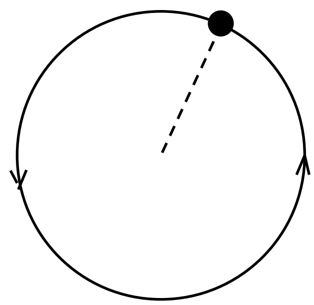
We have learned that any physical body being in motion tends to stay in motion (if the body is at rest it tends to stay at rest). This sentence is called "the law of inertia". It means the following: *velocity* of an object does not change unless the object will interact somehow with other objects. Nothing starts moving by itself and stops by itself. We need another object to change velocity, or, using another words, to create acceleration. The interaction which causes acceleration of an object is called *force*. Force is a vector: it has both magnitude and direction. Examples of the force are: gravity force, electric and magnetic forces, elastic force, friction force.

Next class we will discuss how we can measure force, and what the units of force are. Before we do that let us consider the following imaginary experiment: if you push with the same way an empty shopping cart and a heavily loaded shopping cart. The first one will move faster (check it). A physical quantity which expresses the property of an object to resist acceleration is called *mass*. The mass is measured in kilograms (kg) and grams (g). 1kg=1000g

It is very important not to mix mass and weight. The weight (in common, “everyday” meaning of this word) depends on how strong an object presses to the surface supporting the object. Weight of the same object is different on different planets. The mass express the fundamental property of an object to resist acceleration. Any object with nonzero mass will resist acceleration even in deep space.

Problems (very simple today):

1. Imagine that you rotate a load (say, a metal nut) attached to the end of a thread (see picture below). Suddenly, the thread breaks. Show how the nut will be moving after that.



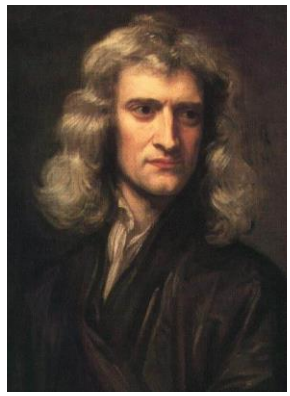
2. What force makes a river flow?
3. Why do we need seatbelts in a car?
4. Imagine that a big meteorite is going to hit the Earth. People are going to change the path of the meteorite and save our planet. What do you believe is the most difficult technical problem people will experience when trying to prevent the collision?

Second Newton’s law

We have learned that force can be determined as interaction which makes the interacting object accelerate. Force and acceleration are connected by a simple formula:

$$\vec{F} = m\vec{a}$$

Here *F* is a force applied to an object, *m* is the mass of the object and *a* is the acceleration of the object. Force is measured in newtons (N). 1N is the force required to provide an acceleration of $1m/s^2$ to an object with a mass of 1kg.The unit of force is named after Sir Isaac Newton (1643-1727)– one of the brightest genii in human history.



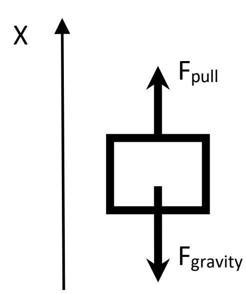
Sir Isaac Newton (www.wikipedia.org)

Arrows over "*F*" and "*a*" remind us that both force and acceleration are vector quantities, which means that they have both magnitude and direction. You can see from the formula that the more mass of the object the more force is needed to provide same acceleration. A heavy object is difficult to accelerate.

However, if an object is not accelerating it does not mean that no forces applied to the object. In most of the cases it just means that forces applied to the object compensate each other. In other words, the sum of all forces applied to the object is zero. So the **force *F* in the formula above is the sum of all the forces applied to the object**. We will call this sum as *total net force*. How we can sum forces?

Example: You pull up a 10kg load with a force of 150N. Is this force enough to lift the load? What is acceleration of the load?

Solution: First, let us make a picture



Let us choose “positive” direction as “down to up”. So the “pulling” force is positive because it looks up and the gravity force is negative because it looks down:

$$F_{pull} - F_{gravity} = ma$$

or

$$F_{pull} - mg = ma$$

We do not know yet what the acceleration (magnitude and sign) is. Let us calculate it:

$$150N - 10kg \cdot 9.8\frac{m}{s^2} = 10kg \cdot a$$

$$a = \left(150N - 10kg \cdot 9.8\frac{m}{s^2}\right) \div 10kg = 5.2\frac{m}{s^2}$$

The acceleration is positive. It means that it is directed up, along our ”positive” axis. It also means that the applied force is enough to lift the load.

Problems:

1. You pull up a 2kg brick with a force of 30N. At a first moment the brick was at rest. Find the displacement of the brick in 3 seconds.

2. You push a loaded shopping cart with a force 100N. The mass of the cart is 40kg. Find the velocity of the cart in 2 seconds if the cart was initially at rest.

3. A 40 kg boy is inside the elevator. Elevator goes up with the acceleration of $2m/s^2$. Find the force which is applied to the boy by the elevator

Third Newton’s law

During last class we have discussed another interesting property of our world. It can be simply formulated as follows: every time one object applies force to another object, this “another” object applies force to the first object. The forces have equal magnitudes and opposite directions.

For example, imagine that you try pushing a heavy 20 kg stone while both you and the stone are on ice. You push the stone with a force of 80N. What happens next?

Both you and the stone will slide in opposite directions. What is acceleration of the stone while you are pushing it? It is simple to calculate it:

$$a_{stone} = \frac{F}{m} = \frac{80N}{20kg} = 4\frac{m}{s^2}$$

What about your acceleration? Assume that your mass is 40kg. But what about force which made you slide? Its magnitude is equal to the magnitude of the force you applied to the stone, but it is directed oppositely. It looks like the stone pushed you with the same force of 80N:

$$a_{you} = \frac{F}{m} = \frac{80N}{40kg} = 2\frac{m}{s^2}$$

So your acceleration is be smaller, because your mass is higher and the magnitude of the forces applied to you and the stone are same.

This “picture” is universal. Whenever you apply force to something this something applies force of equal magnitude and opposite direction to you. *These forces do not compensate each other because they are applied to different objects.* We know that we can add and subtract only the forces applied to the same object.

Now we learned three laws which are the base of simple mechanics:

1. The object in motion tends to stay in motion; an object at rest tends to stay at rest.
2. The total net force applied to an object is equal to the mass of the object multiplied by the acceleration of the object.
3. Any time a force is applied by one object to another, a force of same in magnitude and opposite direction is applied to the first object by the second one.

These laws are called Newton’s laws of motion.

Problems:

1. A compressed coiled spring is released and pushes two carts in opposite directions (see figure below). The carts are different. After the spring is fully stretched the left cart has a velocity of 4cm/s and the right cart has a velocity of 60cm/s. Which cart has higher mass and how many times this mass is higher than the mass of the other cart?

2. Assume that the mass of the right cart is 50g. What is the mass of the other cart? (Use data of the problem 1)

3. Your boat is approaching the riverbank and you are ready to jump to the ground. In what case it will be easier to do that:

a. your boat is empty

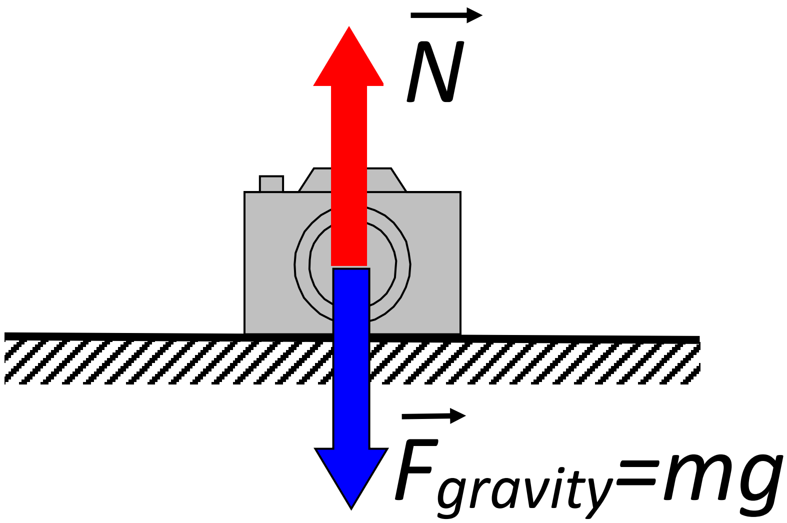
b.your boat is heavily loaded?

Normal force and friction force

Last time we discussed normal force and friction force.

a) Normal force.

Any time we put an object (say, a pen) on a table, floor or any other surface this object apply force to this surface. The origin of this force may be just the gravity (the pen is attracted by Earth). We can also apply additional pressure to the pen. We observe that the pen does not move in vertical direction – it just lies on the table. This means that in spite of the gravity force applied to the pen, the acceleration of the pen in vertical direction is zero. This, in turn, means that the gravity force is compensated by some other force or forces. According to the third Newton's law the surface applies the force of equal magnitude and opposite direction to the object. This force does not allow the pen to go down through the table. We will call this force as "normal force". Normal force is directed perpendicularly to the surface. (Just to remind: two straight lines are called perpendicular if they cross at the right angle. A straight line is called perpendicular ("normal") to the plane if the line is perpendicular to any straight line belonging to the plane)



As we can see in the picture, if the camera just lays on the table, the magnitude of the normal force is equal to the magnitude of the gravity force.

$$ma = N - mg = 0$$

$$N = mg$$

Here our "positive" axis is directed up.

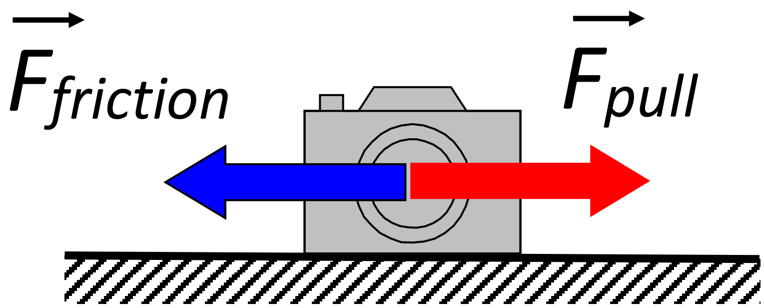
If we will press the camera down with a force F_{press} the normal force will increase to compensate both the gravity force and the pressure force.

$$ma = N - mg - F_{press} = 0$$

$$N = mg + F_{press}$$

b) Friction force *(we will discuss it in detail during next class)*

When we try to push or pull a heavy box standing on the floor it may not move in spite of a considerable pulling of pushing force applied. Some force (or forces) applied to the box by the surface compensates the pushing force and the acceleration in the "pushing" direction is zero. If the magnitude of pushing (or pulling) force is less than certain magnitude which we will call *static friction force*, the box will not move and friction force magnitude is equal to this of the pushing force. If we increase the pushing force, the friction force increases as well until the static friction force is reached. After that, the friction force does not increase anymore and, if we increase the pushing force just a little bit, the box will start moving.



$$ma = F_{friction} - F_{pull} = 0$$

$$F_{friction} = F_{pull}$$

How to calculate the static friction force F_{fs}? The magnitude of the static friction force is proportional to the magnitude of the normal force. Speaking “common sense language” the heavier the box the stronger we have to push to move it.

$$F_{fs} = \mu \cdot N$$

Here μ is the coefficient of friction. This is a number which depends of the object (box) and surface materials and the roughness of the surfaces. If the surfaces are rough, this number is large, so more force is required to move the object.

After the box started moving the friction force is equal to μN. Strictly speaking this is not always correct and, in some cases, the friction force applied to a moving object (dynamic friction force) is not equal to the static friction force. This time we will not discuss this effect in details and, for simplicity, assume that the static friction force is equal to the dynamic one.

Problems:

1. Why there is tread on the surface of a tire?
2. We have learned that any object which is set to motion tends to stay in motion. What about a car? Why can not we just let the car go after the acceleration? Why we have to keep the gas pedal pressed?
3. A 2000kg car accelerates at 5m/s2. The friction coefficient is 1/10. Find the pulling force of the car’s engine.

4. The car from the problem 3 accelerated to 100km/h and moves at a constant velocity. Find total net force applied to the car. :)

Momentum

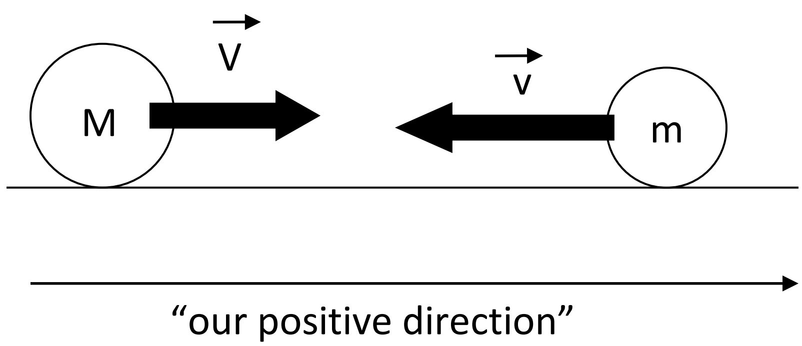
Momentum is the product of the mass and velocity of an object. Momentum is a vector: it has both a direction and a magnitude.

$$\vec{P} = m\vec{V}$$

Momentum is measured in *kg m/s*. As we can see, we can change the momentum by changing the velocity or the mass of an object. To change the velocity we need to apply force. To change the mass of a moving object we could, for example, put an additional load to the rolling cart.

As we can see, if no forces applied to an object (or objects) and total mass of the object (objects) does not change, the momentum does not change as well. In this case we can say that the momentum of the objects *conserves*.

Example: Find total net momentum of two balls with masses *m* and *M* and velocities V and v rolling toward each other.



Solution: First we will choose "positive direction". The momentum and velocity of an object have same direction. I picked up "right-to-left", but the result (as we know) will not change if we choose "left-to-right". The momentum of the left ball is: P=MV. We consider it as positive, because the momentum (as the velocity) "looks" in our "positive" direction. The momentum of the left ball p= - mv. It is negative, because it "looks" in the opposite direction. So the total net momentum, P_{tot} is:

$$P = MV - mv$$

If MV is larger than mv, the total net momentum is positive. It means that it is directed "positively" - left-to-right. If MV is less than mv the total net momentum is negative and "looks" right-to-left.

1. Two cars with mass 1000kg and 2000kg go toward each other. The speed of the first car is 50km/h, the speed of the second is 40 km/h. Find the total net momentum of the two cars. Make a picture.

2. A 80kg jogger runs with a constant acceleration of 1/5 m/s^2 for 10 seconds. How his momentum changed during this time?

3. A 30g pebble falls from the height of 20m. Find its momentum at the moment it hits the ground.

4. A 10kg ball moving at a speed of 10m/s hits a 5kg ball which was at rest before the collision. After the collision the smaller ball starts moving at a speed 10m/s. Find the velocity of the heavy ball after the collision.

5. A fox is chasing a small rabbit. The momentum of the fox is equal to the momentum of the rabbit. Will the fox catch the rabbit?

2. You send a 100g ball up and it returns back in 6 seconds. Find the initial momentum of the ball and its momentum in the highest point.

3. You pushed a loaded shopping cart with the total mass of 50 kg. After the push the cart is rolling (friction is very small) horizontally at a speed 1.4m/s. Then you put another 20kg package into the cart. How will the cart velocity change?

4. Two balls of the same mass ***m*** moving at same velocity ***v*** hit the wall. One ball is made of soft clay – this ball sticks to the wall. Another ball is made of rubber and it bounces back from the wall at the same speed. Compare the momenta transferred to the wall by the balls.

Inpulse

From the second Newton's law we know that we need to apply force to change velocity of an object. Let us consider rectilinear motion (motion along a strait line) of an object with uniform acceleration ***a*** due to constant force ***F***:

$$\vec{a} = \frac{\vec{F}}{m} \quad (1)$$

Where ***m*** is the mass of the object (we assume that the mass does not change). Let us calculate the change in the object's velocity during the time ***t:***

$$\vec{V}_{final} - \vec{V}_{initial} = \vec{a} \cdot t = \frac{\vec{F}}{m} \cdot t \quad (2)$$

Let us multiply both left and right parts of the equation 2 by the mass ***m***:

$$m\vec{V}_{final} - m\vec{V}_{initial} = \vec{F} \cdot t \quad (3)$$

We also know that product of mass and velocity is momentum. So we can write:

$$\vec{P}_{final} - \vec{P}_{initial} = \vec{F} \cdot t \quad (4)$$

If the force F applied during the time t, the product Ft is called ***impulse***. Impulse is a vector. The change of the object momentum is equal to the impulse applied to the object. To understand better formula 4 let us consider a bullet hitting a wall. A bullet has relatively low mass (~10g) but it usually moves at a high velocity (~300-400m/s). We can calculate the momentum of the bullet: P=0.010kg.x 400m/s=4kgm/s. Imagine that the bullet hits a wall and stops in 0.01second. Le us calculate the force applied to the bullet by the wall (I choose the direction of the bullet motion as positive):

$$1) P_{final} - P_{initial} = 0 - 4kgm/s = F \cdot t = F \cdot 0.01s$$

$$2) - 4kgm/s = F \cdot 0.01s$$

$$3) F = -4 \div 0.01 kgm/s^2 = -400N$$

The force is negative – this means that it is directed against the direction of the bullet motion. But from the third Newton's law we know that the force applied by the bullet to the wall has same magnitude but opposite direction. So the answer is 400N.

Problems:

1. A rubber ball (m=10g) is dropped down from the top of 180m building. The ball hits the ground and bounces up with the same speed. Find the force applied by the ball to the ground if the collision time is 0.01s.
2. A soccer ball with m=400g moves at a speed of 25m/s. If the ball will hit the chest of the goalkeeper it bounces back with the same speed and the collision time is 0.025s. If the goalkeeper will catch the ball with his (her) hands, the speed of the ball becomes zero in 0.04s. Find the force applied by the ball to the goalkeeper in both cases.

Energy

Energy is a scalar physical quantity which expresses the ability of an object to do work. It means that an object which possesses some energy can interact with other objects and cause changes in their positions and velocities. An object with higher energy can do more work. There are many forms of energy – kinetic, potential, thermal, chemical nuclear etc. Like momentum, energy conserves. It means that energy can not be created or destroyed – it can only be transferred from one form into another. For example, a power plant does not create electrical energy. The plant converts kinetic energy of water flow or chemical energy of fuel into electrical energy. When a battery of, say, a flashlight is depleted, it does not mean that the energy previously stored in battery just disappeared. The energy was just converted into thermal energy and energy of light.

We start with ***kinetic energy***. The ***kinetic energy*** of an object is the extra energy which it possesses due to its motion. Any moving object possesses kinetic energy. If two objects have same mass, the one with higher speed has higher energy. (I used the word "speed" instead of "velocity" because only the magnitude of velocity is important for the kinetic energy). If the speeds of two moving objects are equal, the object with higher mass will have higher kinetic energy. Kinetic energy can be calculated using the formula:

$$E_{kinetic} = \frac{m \cdot V^2}{2},$$

where ***m*** is mass, ***V*** is speed. The International System unit to measure energy is Joule (J).

$$1J = 1\frac{kg \cdot m^2}{s^2}$$

It is named after James Prescott Joule (1818-1889) – English physicist and brewer.

1. Calculate kinetic energy of a falling stone with a mass of 10kg after 3 second of falling.
2. Imagine that both the mass and the speed of a moving object increased 2 times. How did its kinetic energy change?
3. Calculate kinetic energy of a 2000kg car moving at a speed of 100km/h.

Last class we continued discussing energy, and, in particular the energy of an object due to its motion – kinetic energy. Energy is a scalar. The formula for kinetic energy is

$$E_K = \frac{mv^2}{2},$$

Where *m* is the mass, *v* is the **speed** of the moving object. The word speed is underlined because kinetic energy depends only on the magnitude of the velocity, but not on the direction. Example: a rubber ball bouncing from the wall. Let us assume (this is a realistic assumption) that the magnitude of the velocity (the speed) of the ball does not change after the collision with the wall. The direction of the velocity changes to opposite one.

We know very well that in this case the change in momentum is *2mv (here m is the mass and v is the speed of the ball)*. But the kinetic energy of the ball does not change.

And here is (as I believe) one of major difficulties for understanding:

You can say: "wait a second; we know that the momentum conserves – so if the momentum of the ball changed for 2mv as it bounces back from the wall, the momentum of the wall (and the whole building) is also changed for the same magnitude. If the momentum of the wall is changed - the velocity of the wall will be changed as well. The wall was at rest before collision – so its velocity was zero. Looks like after the collision it will not be zero anymore. But this means that the speed of the wall changes and, hence, its kinetic energy changes too. The wall has to obtain some energy from the ball and speed of the ball has to be reduced after collision."

This is correct.

But let us calculate how the speed of the wall after collision:

Total momentum before the collision is $p_{before} = m_{ball} \cdot v_{ball}$

Total momentum after the collision is $p_{after} = -m_{ball} \cdot v_{ball} + M_{wall} \cdot V_{wall}$

Momentum conservation: $p_{before} = p_{after}$

$$1)\ m_{ball} \cdot v_{ball} = -m_{ball} \cdot v_{ball} + M_{wall} \cdot V_{wall}$$

$$2)\ M_{wall} \cdot V_{wall} = 2m_{ball} \cdot v_{ball}$$

$$3)\ V_{wall} = 2\frac{m_{ball}}{M_{wall}} \cdot v_{ball}$$

We see that the speed of the wall is the speed of the ball multiplied by 2 and by the very very very very small ratio of the ball mass to the wall mass. So, actual speed of the wall is nonzero - but thousand million times less than the speed of the ball. Actually, it is so small that we can forget about it. We also can neglect the kinetic energy obtained by the wall.

If one of the colliding objects has much higher mass than the other, the collision can change the momentum of the light object considerably, but its kinetic energy can stay almost the same. This kind of the collision is called *elastic*. In the result of collision part of the initial kinetic energy of the two balls can be transferred to other energy forms (heat, for example) this kind of collision is called *inelastic*.

Problems:

1. A ball with a mass of 1g moving at a speed 10m/s hits another ball with the mass 100g which was at rest before the collision. After the collision the light ball bounces back with the speed 9m/s. Find kinetic energy of the heavy ball after the collision. (Suggestion: use momentum conservation to find the speed of the heavy ball then calculate its kinetic energy).
2. Calculate the kinetic energy which was lost by the light ball after the collision. Compare it with the kinetic energy obtained by the heavy ball and try to explain the result.
3. Where goes the kinetic energy of a car after the driver presses the brake pedal and the car stops? What happens with the momentum of the car?

To solve the problems 1 and 2 you have to use both the momentum conservation and the energy conservation laws. We assume that all the kinetic energy which is lost after the collisions is transferred to heat.

1. Two led balls are rolling towards each other. One ball has a mass of 1kg and a speed of 20m/s; the other – 2kg and 4m/s. After collision the balls roll together. Find the amount of energy which was transferred to heat.
2. A bullet of a mass 10g breaks through a 1kg box with sand standing at a frictionless surface. Before the collision the speed of the bullet was 300m/s. After the bullet vent through the box the speed of the bullet was 200m/s. Find the amount of energy which was transferred to heat.
3. Solve problem 3 for the case the bullet is trapped in the box.

Potential energy

We can think about potential energy as of the energy "stored" in the system. Unlike kinetic energy which depends on the object's *velocity*, potential energy depends on the *position* of the physical body with respect to other objects with which the body interacts. Expression for potential energy depends on the type of interaction between the objects. Here we discuss how to calculate the potential energy in case of gravity force. Any object with mass is attracted by Earth. The higher is the position of the object over the ground level the stronger it will hit the ground when it falls. It is natural to assume that potential energy depends on the distance between the object and the ground. When a stone starts falling it accelerates toward the Earth. The kinetic energy of the stone increases. At a first glance it looks like the energy is created as the stone goes down. But this statement is not correct. The total energy of the stone remains constant as the stone is falling down, in full agreement with the energy conservation law. The total energy of the stone is the sum of potential and kinetic energies. In the highest point kinetic energy is zero and potential energy is maximal. At the lowest point, just before the stone hits the ground, potential energy is minimal and kinetic energy is maximal. Potential energy corresponding to the gravity force can be calculated as

$$E_{potential} = m \cdot g \cdot h$$

Here *m* is the mass of the object; *g* is acceleration due to gravity, *h* is the distance between the object and, say, the ground. There are two important points:

1. As with the kinetic energy absolute value of potential energy does not make much sense. We can count h from any level when solving a problem. What does make sense it is *change* in the potential energy in a certain process. This change will not depend on the "zero potential energy level" which you can choose arbitrary.
2. The formula above is valid only if the object is close enough to the Earth – the distance between the object and the Earth should be much less than the radius of Earth.

1. A 1 kg stone is falling down from a height of 10m. Calculate kinetic and potential energies of the stone in the upper, middle and lower points. Please calculate the kinetic energy through the calculation of the stone's velocity and show that the total energy of the stone remains constant as it goes down.
2. A 10g bullet is sent up at a speed of 300m/s. How high it will go? Solve this problem by two ways.
1. Imagine that you climb up a 10m tree. Calculate how does your potential energy with respect to the ground change?
2. You keep a 1kg book in your stretched hands right over your head. Calculate potential energy of the book with respect to the floor level and with respect to the top of your head.
3. A 50g ball is falling down. As the ball passes a certain distance its potential energy changes for 2J. Calculate this distance. Does this distance depend on the initial velocity of the ball?

Mechanical work

Last time we learned that in order to change kinetic energy of an object we have to change the absolute value of its velocity (the speed). To do that we need acceleration and, hence, ***force*** (we know very well that there is no acceleration without force). But it is not enough just to apply force. We also know that there is acceleration which just changes the direction of the velocity but leaves its absolute value unaltered. For example when you ride on merry-go-round your speed is constant as well as kinetic energy. But this is accelerated motion since the direction of your motion is changing permanently. To maintain this kind of motion we need force (this force is called *centripetal force* – we will discuss it later).

To change kinetic energy of an object we should have the ***displacement*** of the object ***along the direction of the force***. If there are both force applied to an object and the displacement of the object along the force a physicist says that ***the force performs the work on the object***. The "everyday" meaning of the word "work" is different (for example, a swimming pool guard may spend the whole working day watching the swimming people – there is no force, and, sometimes, no much of displacement☺). In physics the work is:

Work = **Force** x **Displacement, parallel to the force**

A=Fs

To change kinetic energy of an object we have to apply force which will perform work on the object. In spite of the force and the displacement are vectors, the work is a scalar – it has no direction, just magnitude.

<u>The work neither creates nor destroys the energy. This is impossible. But the work is the tool which converts the energy from one form to another and/or transfers the energy from one object to another</u>.

The unit for the work measurement in the International System is Joule (J) – same as for the energy.

The work can be positive and negative work. When the object moves along the applied total net force the work is positive and the kinetic energy of an object increases. Sometimes the force applied against the displacement of an object – in this case the object decelerates, the work of the force is negative in this case and the kinetic energy decreases. The work not always changes the kinetic energy of the object, but ***to change the kinetic energy the object work has to be done***.

<u>Example 1</u>: We push a cart of the mass ***m*** with the force ***F.*** At the first moment the cart was at rest. Find the kinetic energy of the cart after it passes distance ***d***.

<u>A possible way of solution:</u>

1. Let us find the acceleration of the cart: $a = \frac{F}{m}$

2. Let us find the time ***t*** which is necessary to cover the distance ***d***:

$d = \frac{at^2}{2}$, so we can find ***t***: $t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm}{F}}$

3. Let us find the speed of the cart in the time ***t***: $v = at = \frac{F}{m} \cdot \sqrt{\frac{2dm}{F}}$

4.Finally, let us calculate the kinetic energy:

$$E_K = \frac{mv^2}{2} = \frac{1}{2} \cdot m \cdot \left(\frac{F}{m}\right)^2 \cdot \left(\sqrt{\frac{2dm}{F}}\right)^2 = \frac{1}{2} \cdot \frac{m \cdot F \cdot F \cdot 2 \cdot d \cdot m}{m \cdot m \cdot F} = F \cdot d \;\; !!!$$

<u>Another possible way of solution:</u> $E_K = F \cdot d$ .This is simpler. We just have to know that the work performed on the cart went to increase its kinetic energy.

<u>Example 2:</u> At the beginning we push the cart of the mass ***m*** and it started moving at a speed of v_0**.** How far it will roll if the friction force is ***F***?

<u>A possible way of solution:</u>

1. Let us find the acceleration of the cart: $a = -\frac{F}{m}$ ("minus" is because the acceleration due to friction is directed against the motion)

2. Let us find in what time ***t*** the cart will stop:

$$t = \frac{v_{final} - v_0}{a} = \frac{0 - v_0}{a} = -\frac{v_0}{a} = \frac{mv_0}{F}$$

4.Finally, let us calculate the distance ***d***:

$$d = v_0 t - \frac{at^2}{2} = v_0 \cdot \left(\frac{mv_0}{F}\right) - \frac{1}{2}\left(-\frac{F}{m}\right) \cdot \left(\frac{mv_0}{F}\right)^2 = \frac{mv_0^2}{F} - \frac{1}{2}\frac{mv_0^2}{F} = \frac{mv_0^2}{2} \div F$$

$$d = \frac{E_K}{F}$$

<u>Another possible way of solution:</u>

$E_K = F \cdot d$. The friction force performs negative work on the cart and "eats "kinetic energy of the cart. I said "eats" but we know that the energy can not be destroyed. Friction just transforms the kinetic energy into another energy form – heat.

So $d = \frac{E_K}{F} = \frac{mv_0^2}{2} \div F$

Problems:

1. A 1kg rock falls down from the height of 10m. Find the kinetic energy of the rock at the moment it hits the ground.
2. Solve the problem 1 by a different way.
3. Find the work of the friction force which is necessary to stop the 1000kg car moving at a speed of 72km/h. The friction coefficient is 0.1
4. A barrel is full of water. A boy emptied half of the barrel using a bucket. A girl emptied the other half. Who performed more work: the boy or the girl? Or, may be the works were equal?
5. A water pump lifts 20 liters of water per second to the water supply tank which is 10m over the ground level. What work is performed by the pump per 1 hour?