Math 3 Classwork 22

Warm Up

1 Multiplication and Division Quiz. Do as many problems as you can in **5 minutes.**

Independent work

50 × 0 =	30 × 15 =	60 × 15 =
30 × 5 =	4 × 300 =	400 × 4 =
60 × 90 =	60 × 200 =	60 × 400 =
8 × 10 =	80 × 10 =	80 × 50 =
2 × 75 =	20 × 75 =	2 × 65 =
9 × 90 =	9 × 50 =	90 × 30 =
70 × 6 =	80 × 70 =	70 × 7 =

100 ÷ 2 =	24 ÷ 4 =	25 ÷ 5 =
160 ÷ 8 =	160 ÷ 20 =	300 ÷ 150 =
300 ÷ 60 =	350 ÷ 50 =	1000 ÷10 =

2 Calculate 2 + 2 - 2 + 2 - 2 + 2 - 2 + 2 - 2 + 2

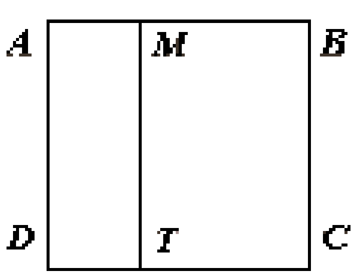
A. *0* ***B.*** *2* ***C.*** *4* ***D.*** *12* ***E.*** *20*

3 The human heart beats approximately 70 times per minute. How many beats approximately will it make in an hour?

A. *42 000* ***B.*** *7 000* ***C.*** *4 200* ***D.*** *700* ***E.*** *420*

4 *ABCD* is a square. Its side is equal to 10cm. *AMTD* is a rectangle. Its shorter side is equal to 3 cm. How many centimeters is the perimeter of the square *ABCD* larger than that of the rectangle *AMTD*?

A. *14 cm.* ***B.*** *10 cm.* ***C.*** *7 cm.* ***D.*** *6 cm.* ***E.*** *4 cm.*



Homework Review

5 Find the 3-digit numbers, where the digit in its ten places is twice the digit in its hundreds place. The digit in its one place is 4 times the digit in its hundreds place. Write down all numbers that satisfy these conditions.

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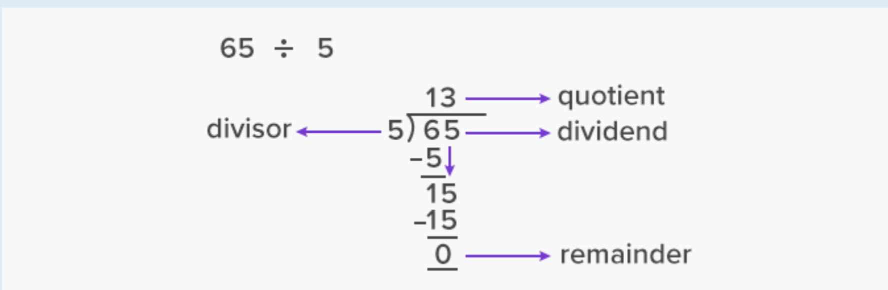
6 Postal workers must not lift more that 16 kg. There are 6 packages with a different weight: 3100g, 4900g, 3950g, 7200g, 3700g and 8900g.
Arrange these packages into 2 piles so that each pile can be lifted safely.

1) __________________________

2) __________________________

REVIEW I

In math, **long division** is a method used for dividing large numbers into groups or parts.
Long division helps in breaking the division problem into a sequence of easier steps.
Just like all division problems, a large number, which is the **dividend**, is divided by another number, which is called the **divisor**, to give a result called the **quotient** and sometimes a **remainder**.



7 Long division:

a) 243 ÷ 3 =

b) 576 ÷ 8 =

Independent work

In the set theory, the elements (or members) are collected on the basis of one or more common properties to form a set. So, each element is a member of that set. Hence, it is simply expressed as the element belongs to the set.
An Italian mathematician, Giuseppe Peano used a Greek letter lunate epsilon ($\in$) for expressing the phrase "belongs to" symbolically in set theory. It helps us to express the relationship between an element and its set in mathematical form.

8

a) Consider the set A on the number line, which includes all whole numbers between 1 and 10. Use the symbols defining "is an element of" or "is not the element of" to answer the following questions.

Q1: Does number 6 belong to the set A? _______________

Q2: Does number 1 belong to the set A? _______________

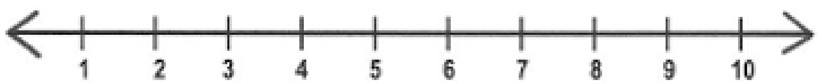
Q3: Does number 11 belong to the set A? _______________

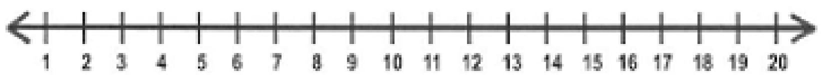
a) Consider the set B on the number line, which includes all whole numbers between 1 and 20. Use the symbols defining "is an element of" or "is not the element of" to answer the following questions.

Q1: Does number 6 belong to the set B? _______________

Q2: Does number 1 belong to the set B? _______________

Q3: Does number 21 belong to the set B? _______________





New Material I

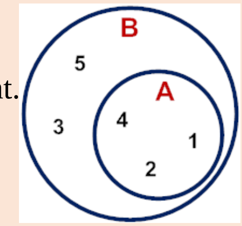
A subset is a set whose elements are all members of another set.

Example 1: Given $A = \{1, 2, 4\}$ and $B = \{1, 2, 3, 4, 5\}$, what is the relationship between these sets?

We say that A is a subset of B, since every element of A is also in B. This is denoted by:

$$A \subset B$$

A Venn Diagram for the relationship between these sets is shown to the right.



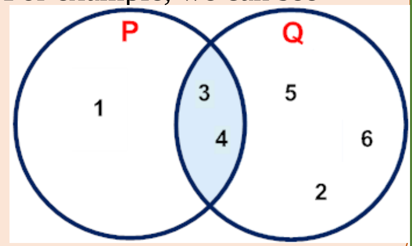
Answer: A is a subset of B.

*Example 2***:** Given $P = \{1, 3, 4\}$ and $Q = \{2, 3, 4, 5, 6\}$, what is the relationship between these sets?

We say that P is not a subset of $\mathbf{Q}$ since not every element of P is contained in Q. For example, we can see that $1 \notin Q$. The statement *"P is not a subset of Q"* is denoted by:

$$P \not\subset Q$$

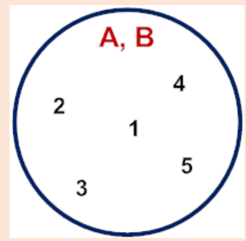
Note that these sets do have some elements in common.



The intersection of these sets is shown in the Venn diagram to the right.

Example 3: Given $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 1, 2, 5, 4\}$, what is the relationship between A and B? Recall that the order in which the elements appear in a set is not important. Looking at the elements of these sets, it is clear that:

$$A \subset B$$
$$B \subset A$$
$$A = B$$



Answer: *A* and *B* are equivalent.

9 Compare the sets A and B. A = {♔ ♕ ♖ ♗} B = { ♕ ♖ ♘ ♔}

What is the element of the set A that is not included in set B? __________

What is the element in the set B that is not included in set A? __________

Write down all elements of the set C, which is C = A ∩ B ______________________

Write down all elements of the set D, which is C = A ∪ B ______________________

Write down all elements of the set E, which is E = A ⊂ B ______________________

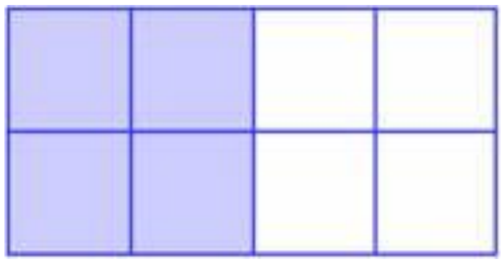
New Material II

Many problems in mathematics deal with **whole numbers**, which are used to count whole units of things. For example, you can count students in a classroom and the number of dollar bills.

You need other kinds of numbers to describe units that are not whole. For example, an aquarium might be partly full. A group may have a meeting, but only some of the members are present.

Fractions are numbers used to refer to a part of a whole. This includes measurements that cannot be written as whole numbers. For example, the width of a piece of notebook paper is more than 2 cm but less than 3cm. The part longer than 2cm is written as a fraction. Here, you will investigate how fractions can be written and used to represent quantities that are parts of the whole.

A whole can be divided into parts of equal size. In the example below, a rectangle has been divided into eight equal squares. Four of these eight squares are shaded.

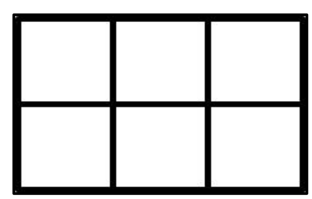


The shaded area can be represented by a fraction. A fraction is written vertically as two numbers with a line between them. $\frac{1}{2}$

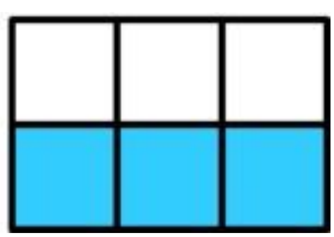
The **denominator** (the bottom number) represents the number of equal parts that make up the whole. The **numerator** (the top number) describes the number of parts that you are describing

Looking into the example above, the rectangle has been divided into 8 equal parts, and 4 of them have been shaded. You can use the fraction $\frac{4}{8}$ to describe the shaded part of the whole.
In $\frac{4}{8}$, the 4 is the **numerator** and tells how many parts are shaded. The 8 is the **denominator** and tells how many parts are required to make the whole.

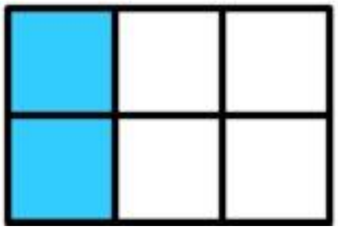
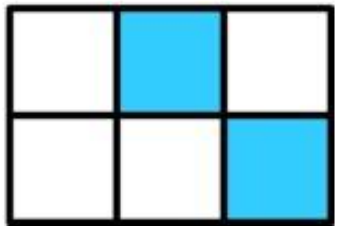
10 a) Each small square is a square unit. What is the area of this rectangle?
A = ________ square units.

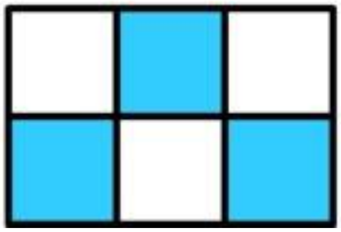
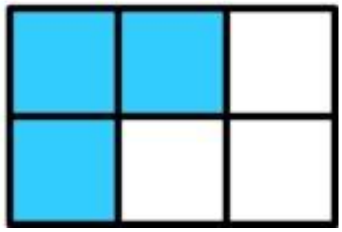


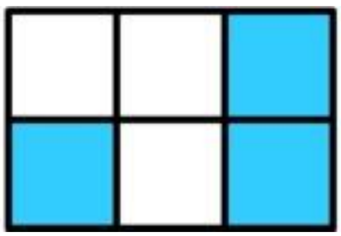
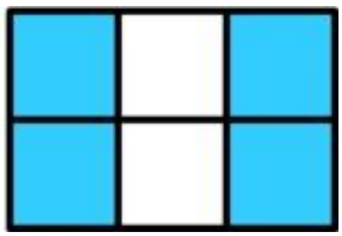
b) What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.

A.  B. 

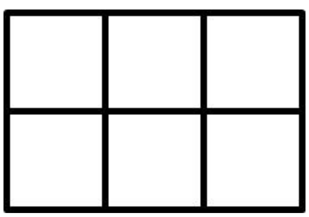
A: ______, B: __________

C.

D.

C: _______, D: ________

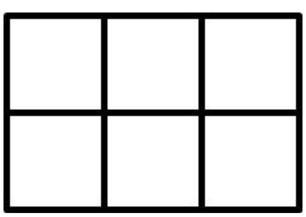
E.

F.

E: ______, F: _______

G.

H.

G: ______, H: ________

c) Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.



d) Shade $\frac{2}{3}$ of the area of the rectangle in a way that is different from the rectangles above.



11 There are total of 11 children in the class.
2 out of 11 are girls. Write the fraction that will illustrate the number of girls ____________

9 out of 11 are boys. Write the fraction that will illustrate the number of boys ___________

Did you know …

Why do we study set theory?

We like to think that mathematics developed from the need of our ancients to count things. I have four sheep, you have sixteen camels, my tribe has ten dozen of men, you have six hundred wives... etc. etc. But if you look closely, counting how many things you have of a certain type, first required you have been able and collect them into one collection. The "collection of all sheep I have", or the "collection of men in my tribe", and so on.

Sets came to solve a similar problem. Sets are collections of mathematical objects which themselves are mathematical objects.

This, of course, doesn't mean that we should learn set theory just for that purpose alone. The applications of set theory are not immediate for finite collections, or rather sufficiently small collections. We don't need to think about pairs or sets with five elements as particular objects. Whatever we want to do with them we can pretty much do by hand.

Sets come into play when you want to talk about infinite sets. Infinite sets collect infinitely many objects into one collection. The set of natural numbers, the set of finite sets of sets of sets of natural numbers, the set of sets of sets of sets of sets of sets of irrational numbers, etc. Once you establish that mathematical objects can be collected into other mathematical objects you can start analyzing their structure.

But here comes the problem. Infinite sets defy our intuition, which comes from finite sets. The many paradoxes of infinity which include Galileo's paradox, Hilbert's Grand Hotel, and so on, are all paradoxes that come to portray the nature of infinity as counterintuitive to our physical intuition.

Studying set theory, even naively, is the technical spine of how to handle infinite sets. Since modern mathematics is concerned with many infinite sets, larger and smaller, it is a good idea to learn about infinite sets if one wishes to understand mathematical objects better.

And one can study, naively, a lot of set theory, especially under the tutelage of a good teacher that will actually teach axiomatic set theory in a naive guise. And this sort of learning can, and perhaps should, include discussions about the axiom of choice, about ordinals, and about cardinals. As Ittay said, and I'm agreeing ordinals and cardinals are two ways of counting, which extend beyond our intuitive understand that counting is done via the natural numbers and allow us to count infinite objects.

If one couples these ideas with the basics of first-order logic, predicate calculus, and basic first-order logic, one understands how set theory can be used as a basis for modern mathematics. Which again, allows us to better see into some parts of mathematics.

Set theory is a fundamental concept throughout all of mathematics. This branch of mathematics forms a foundation for other topics. In the areas pertaining to statistics, it is particularly used in probability. Much of the concepts in probability are derived from the consequences of set theory.