Math 4a. Classwork 2.

Addition | **Multiplication**

Commutative property

$$a + b = b + a$$

$$a \times b = b \times a$$

Associative property

$$a + (b + c) = (a + b) + c$$

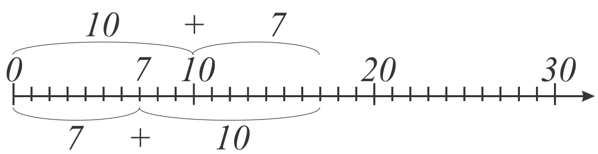
$$a \times (b \times c) = (a \times b) \times c$$

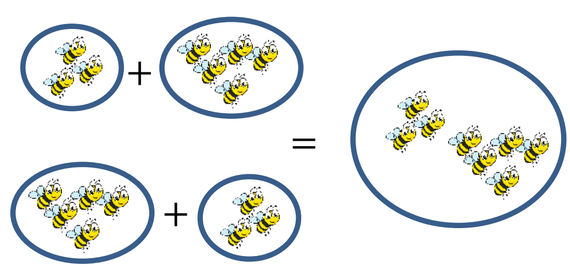
Distributive property

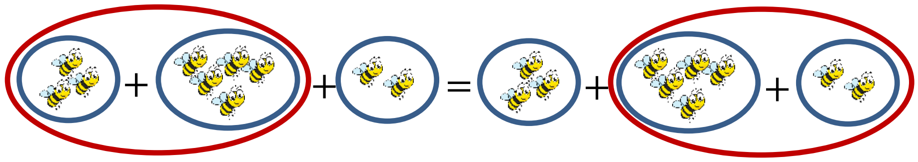
$$a \times (b + c) = a \times b \ + \ a \times c$$

$$a \times (b - c) = a \times b - a \times c$$

Commutative and associative properties of addition are intuitively easy to understand.

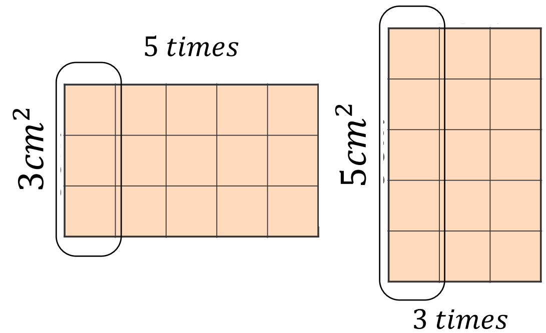






The commutative property of multiplication can be illustrated by calculating the area of a rectangle (see example on the picture):

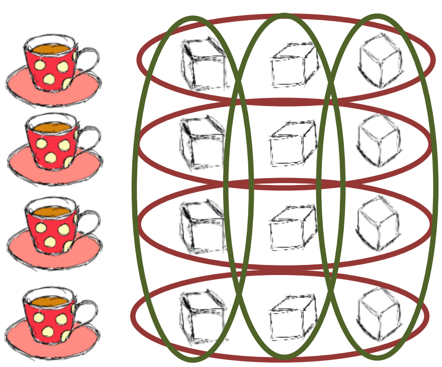
$$S = 3cm^2 \times 5times = 5cm^2 \times 3times$$
$$= 15cm^2$$



Another example of the commutative property of multiplication:

How many sugar cubes do you need for four cups of tea if three cubes are enough for one cup?

There are two ways to calculate the number of sugar cubes:



- Put three cubes into each cup

$$3\ cubes \cdot 4 = 12\ cubes$$

- Or put one cube into each cup, then the second to each cup, then the third

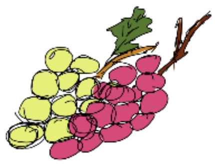
$$4\ cubes \cdot 3 = 12\ cubes$$

Of course, the results are the same, 12 cubes of sugar are needed for 4 cups of tea.

The following problem can show the distributive property:

The farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of grapes (green and red) did the farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes?

We can first find out how many boxes of grapes the farmer has and multiply it by 5lb (in each box), or we can find out the weight of white and red grapes and then add it.



$$5 \times (10 + 8) \quad or \quad 5 \times 10 + 5 \times 8$$

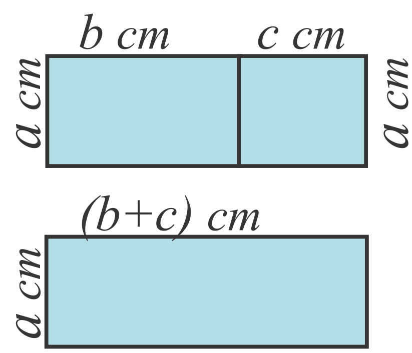
$$5 \times 18 = 50 + 40; \qquad 90 = 90$$

Another example:
The combined area of these two rectangles is

$$S = a \times b + a \times c$$

The rectangle with one side a cm and the other side $(b + c)$ cm will have exactly the same area

$$S = a \times b + a \times c = a \times (b + c)$$

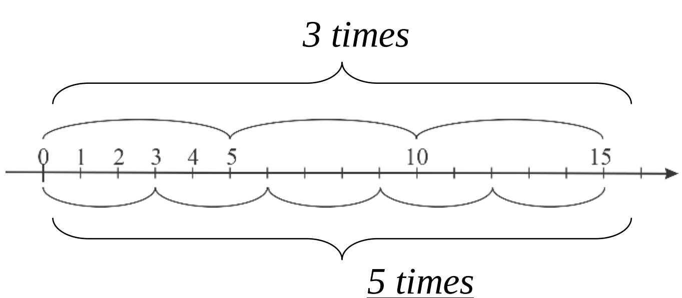


We discussed a few properties of addition and multiplication. As we all know, multiplication is an arithmetic operation, equivalent to the repetitive addition of the same number.

$$c \times b = \underbrace{c + c + c + \cdots + c}_{b\ times} = \underbrace{b + b + b + \cdots + b}_{c\ times} = a$$

The result of a multiplication is called a *product*, the participants of the operation are called *factors*. c and b are factors, and a is a product.

Multiplication is connected with division; when we are dividing a number (this number is called a *dividend*) by a *divisor*, we are looking for a number (a *quotient*), such that gives us a dividend if multiplied by a divisor.



(In this part of our course, we're talking about natural numbers, the numbers that we use for counting, starting with 1: 1, 2, 3…. I'll just use the word "number" and omit the word "natural".)

If there is a number c, that $c \times b = a$, then we can say that $a \div b = c$, a is divisible by b, b can be “fit” into a whole number of times. c also is a factor of a, $a \div c = b$. For example,

$$3 \times 5 = 15; \quad 15 \div 3 = 5, \quad 15 \div 5 = 3$$

factor *factor* *product*

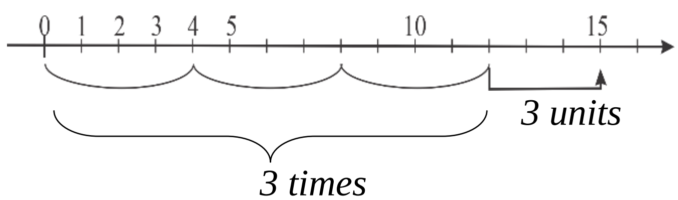
$$5 \cdot 3 = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = 15$$

5 can fit into 15 exactly 3 times, 3 can go into 15 exactly 5 times.
15 is divisible by 3 and by 5.

If there is no such number, then we can say that this number is not divisible by the divisor. But in this case, we can use division with a remainder.
For example, $15 \div 4$. 4 can't fully complete 15. It can fit into 12 three times, but there will be a little more left. So, $15 \div 4 = 3R(3)$, or $15 = 4 \times 3 + 3$

a:b=c
dividend *divisor* *quotient*

a=b·c+r
dividend *divisor* *quotient* *remainder*



For division of any natural number by another, we can now write:

$$a \div b = cR(r), or\ a = b \times c + r$$

If $r = 0$, number a is divisible by number b.

Why can't you divide by 0? By definition, multiplying 0 with something is 0. Dividing by 0 means that there is a number that can be multiplied by 0, and the result will not be 0. But this is impossible. So, division by 0 is not defined, there is no such thing; we can't do that!

Divisibility rules.

Can we predict whether a given number is divisible by 2, 3, 4, and so on? There exist the following divisibility rules.

1. ***A number is divisible by 2 if and only if its last digit is even or 0.***

2. ***A number is divisible by 3 if and only if sum of its digits is divisible by 3.***

3. ***A number is divisible by 4 if and only if the number formed by the last 2 digits is divisible by 4.***

4. ***A number is divisible by 5 if and only if its last digit is 5 or 0.***

5. ***A number is divisible by 6 if it is divisible by 2 and 3 at the same time, so it will be divisible by 6 if an only if its last digit is even or 0 and the sum of its digits is divisible by 3.***

Examples of the solving problems:

Even or odd number is the sum of four odd numbers?

Any odd number can be written as an even number plus 1.
So, if four odd numbers are added together, one can say that it's a sum of four even numbers plus 4. Each even number has a factor 2, 4 also can be written (factorized) as a product of $2 \cdot 2$*. They all have common factor 2, it can be factored out (distributive property), and the sum will be represented as a product of 2 and the sum of several numbers. So, it will be even number.*

Without calculating, establish whether the product of 625 and 18 is divisible by 3?
$625 \cdot 18 = 625 \cdot 3 \cdot 6$
The product has factor 3, so it is divisible by 3.

Evaluate (what is the best way to compute it? Hint: use the distributive and/or commutative property):

$113 \cdot 12 - 12 \cdot 13 = 12 \cdot (113 - 13) = 12 \cdot 100 = 1200$

$135 + 248 - 35 + 52 = 135 - 35 + 248 + 52 = 100 + 300 = 400$

Divide with remainder:
$78 \div 12$;

The closest smaller number, divisible by 12 is 72: $12 \times 6 = 72$, *the remainder is* $78 - 72 = 6$. *Check:*

$$6 \times 12 + 6 = 78$$