Math 4a. Classwork 3.

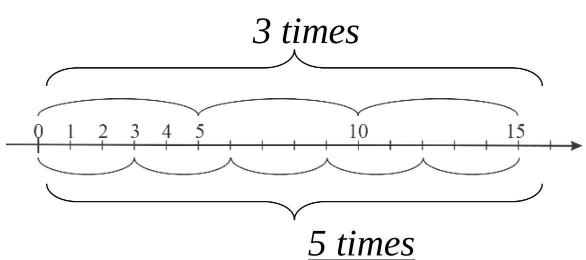
The result of multiplication is called *product*, and the participants of the operation are called *factors*. c and b are factors, and a is a product.

Multiplication is related to division; when we are doing division of a number (the *dividend*) by a *divisor*, we are trying to find a number (a *quotient*), that produces the dividend if multiplied by the divisor.

(In this part of our course, we are talking about natural numbers, the numbers that we use for counting, starting from 1: 1, 2, 3…. I will omit the word "natural" and use only the word "number".)

If there is a number c, that $c \times b = a$, then we can say that $a \div b = c$, a is divisible by b, b can be “fit” into a whole number of times. c also is a factor of a, $a \div c = b$. For example,

$3 \times 5 = 15; \quad 15 \div 3 = 5, \quad 15 \div 5 = 3$



factor factor product

$5 \cdot 3 = 5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = 15$

5 can fit into 15 exactly 3 times, 3 can go into 15 exactly 5 times.
15 is divisible by 3 and by 5.
(We can divide equally 15 candies between 5 kids, between 3 kids).
If there is no such number that the divisor enters the dividend several times, then we can say that this number is not divisible by the divisor. But in this case, we can use division with a remainder.

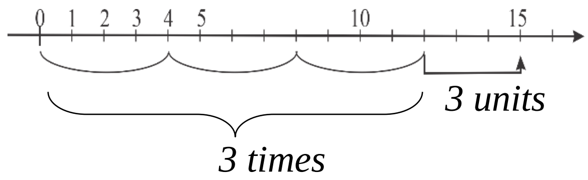
$a:b=c$

dividend divisor quotient

$a=b\cdot c+r$

dividend divisor quotient remainder

We can’t divide 15 candies equally between 4 kids.
$15 \div 4$. 4 can't fully complete 15. It can fit into 12 three times, but there will be a little more left (3 to be exact). So, $15 \div 4 = 3R(3)$, or $15 = 4 \times 3 + 3$



For division of any natural number by another, we can now write:

$$a \div b = cR(r), \qquad or \qquad a = b \times c + r$$

If $r = 0$, number a is divisible by number b.

Why can't you divide by 0? By definition, multiplying 0 with something is 0. Dividing by 0 means that there is a number that can be multiplied by 0, and the result will not be 0. But this is impossible. So division by 0 is not defined, there is no such thing, we can't do that!

Divisibility rules.

Can we predict whether a given number is divisible by 2, 3, 4, and so on? There exist the following divisibility rules.

1. ***A number is divisible by 2 if and only if its last digit is even or 0.***
Let’s try to see why it’s true.
Any natural number can be written in an extended form:
$$\ldots + 100 \cdot (umber\ of\ hundereds) + 10 \cdot (number\ of\ tens) + number\ of\ unites$$
If the last digit, number of units, is even, then each term has a common factor 2 and this factor can be factor out of the parenthesis, and number will have a factor 2, so, will be divisible by 2. If it’s 0, we don’t have any units, number is divisible by 10, and 10 is a product of 2 and 5.

2. ***A number is divisible by 3 if and only if sum of its digits is divisible by 3.***

3. ***A number is divisible by 4 if and only if the number formed by the last 2 digits is divisible by 4.***

4. ***A number is divisible by 5 if and only if its last digit is 5 or 0.***

5. ***A number is divisible by 6 if it is divisible by 2 and 3 at the same time, so it will be divisible by 6 if an only if its last digit is even or 0 and the sum of its digits is divisible by 3.***

6. ***A number is divisible by 9 if and only if sum of its digits is divisible by 9.***

Exercises. Problems with * are more difficult.

1. Find all divisors of numbers 6, 7, 14, 18, 70.

2. If we want to divide a number by 9, what numbers can we get as a remainder?

3. Fill in the empty cell in the table:

dividend	a	29		46	94
divisor	b	7	9		9
quotient	c	4	7	3	
remainder	r		5	1	4

Check the formula $a = b \cdot c + r$ for each number in the table.

4. 85 and 187 are both divisible by 17. Will their sum by divisible by 17?

5. The remainder of $1932 \div 17$ is 11, the remainder of $261 \div 17$ is 6. Is
$$2193 = 1932 + 261$$
divisible by 17? Is it possible to say without division?

6. Find all natural numbers such that when divided by 7, the quotient and remainder are equal?

7. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number
      Can you explain why?

8.
   a. Will the following numbers be divisible by 2:

      123457, 1029384756, 43567219874563157830

   b. by 3
      1347, 45632, 5637984265
   c. by 5:
      5635, 78530, 657932, 45879515

9. Is the product of 1247 and 999 divisible by 3 (no calculations)?

10. Number a is divisible by 5. Is the product $a \cdot b$ divisible by 5?

11. Without calculating, establish whether the product is divisible by a number?

   $a.$ $508 \cdot 12$ by 3 $\qquad$ $b.$ $85 \cdot 3719$ by 5
   $c.$ $2510 \cdot 74$ by 37 $\qquad$ $d.$ $45 \cdot 26 \cdot 36$ by 15
   $e.$ $210 \cdot 29$ by $3, by$ 29 $\qquad$ $f.$ $3800 \cdot 44 \cdot 18$ by $11, 100, 9$

12. Without calculating, establish whether the sum is divisible by a number:
   $a.$ $25 + 35 + 15 + 45$ by $5;$ $\qquad$ $b.$ $14 + 21 + 63 + 24$ by 7
   $c.$ $18 + 36 + 55 + 90$ by $9;$

13. How many vans are needed to take 55 students on a field trip if a van can take 12 students?

14. The summer vacation is 73 days long. Which day of the week will be last day of vacations if the first day was Tuesday?

15. Show that among any three consecutive natural numbers there will be one divisible by 3.

16. Among four consecutive natural numbers will be a number

    a. Divisible by 2?
    b. Divisible by 3?
    c. Divisible by 4?
    d. Divisible by 5?

17. Which statement is true and which is false:

    a. If number is divisible by 9, it's divisible by 3.
    b. If number is divisible by 3, it's divisible by 9.

18. There are 24 students in the class. How they can be divided into equal teems?

19. Give an example of a number which gives the remainder 1 when divided by 2 and by 3.

20. Give an example of a number which gives the remainder 1 when divided by 2 and and remainder 2 when divided by 3.

21. *If pencils are put into boxes, 8 pencils in a box, then 5 extra pencils will remain. If 6 pencils are put in each box, then there will also be 5 extra pencils. How many pencils are there if there are more than 50 but less than 100?

22. *Can you tell without calculating, what will be a remainder when

    a. 137 is divided by 10, by 5, by 3?
    b. 543 is divided by 2, by 5, by 9?