Math 4a. Classwork 4.

Factorization.

In mathematics factorization is a decomposition of a number or mathematical expression as a product of numbers or/and expressions.

A number can be represented as the product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 2 \cdot 3 \cdot 6$$

Numerical expression can be written as a product:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Is it possible for any natural number to be expressed as a product of 2 or more numbers other than 1 and itself?

Natural numbers, greater than 1 that has no divisors other than 1 and itself are called **prime numbers**.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization (or integer **factorization)** is a decomposition of a natural number into the product of prime numbers.

Prime factorization process:

168	2
84	2
42	2
21	3
7	7
1	

180	2
90	2
45	3
15	3
5	5
1	

Prime factors of 168 are $2, 2, 2, 3, 7$, and prime factors of 180 are $2, 2, 3, 3, 5,$

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168; \quad 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

Any natural number has single unique prime factorization.

For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummi worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly?

To solve this problem, we have to find a number which can serve as a divisor for 168 as well as for 180. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor. The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this they have to find the Greatest Common Divisor (GCD), the largest number that can be a divisor for both (168 and 180) amounts of candy. Let's take a look at a set of all prime factors of 168 and 180. For 168 this set contains 2, 2, 2, 3, and 7. Any of these numbers, as well as any of their products can be a divisor for 168. The same is true for the set of prime factors of 180, which are 2, 2, 3, 3, and 5. It is easy to see that these two representations have common factors, 2, 2, 3. It means that both numbers are divisible by any of these common factors and by any of their products. The largest product is the product of all common factors. This largest product has a name: Greatest Common Factor, or GCF. This GCF will also be a Greatest Common Divisor (GCD). $\mathrm{GCF}(168, 180) = 12$

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$$

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

$$168 \div 12 = 14$$

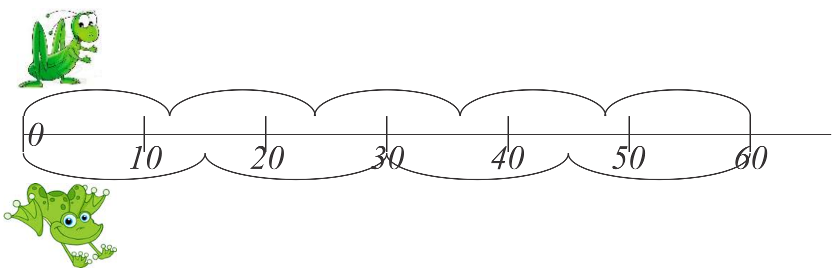
$$180 \div 12 = 15$$

Between 12 kids they can divide both kinds of candy evenly.

A grasshopper jumps a distance of 12 centimeters each jump. A little frog leaps a distance of 15 centimeters each time. They both start at 0 and hop along the long ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after a certain number of jumps. One of such places is, of course, cm. A grasshopper would make 15 jumps, while a frog would do only 12. Will be the only place where they can meet or are there other places?

Any multiple of will also be divisible by 12 and by 15. Are there any other common multiples of 12 and 15, which are less then 12×15 and are still divisible by 12 and 15?



Prime factorization of 12 and 15:

12	2	15	3
6	2	5	5
3	3	1	
1			

$$12 \cdot 15 = (2 \cdot 2 \cdot 3) \cdot (3 \cdot 5)$$

The number which we are looking for has to be a product of prime factors of 12 and 15.

$$2 \cdot 2 \cdot 3 = 12$$

$$3 \cdot 5 = 15$$

$$2 \cdot 2 \cdot 3 \cdot 5 = 60$$

60 is the smallest number, which is divisible by 12 and 15, LCM.

$$\text{LCM}(12, 15) = 60$$

The Johnson family wants to buy the same number of gammy worms and mini chocolates (168 mini per box and 180 gammy worms per box). How many boxes of each type of candy do they need to buy?

$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 168$$

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2520$$

If all factors from one number are multiplied by the factors from the second number, which are missing in the first one, the resulting number is divisible by both numbers (is a common multiple) and is the smallest common multiple.

They need to buy $2520 \div 168 = 15$ boxes of mini chocolates and

$2520 \div 180 = 14$ boxes of gammy worms. Least common multiple for 168 and180 is 2520, which I much smaller then $168 \cdot 180 = 30240$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in mathematics as the Sieve of Eratosthenes.
In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Exercises.

1. Do the prime decomposition of the numbers: 66, 28, 128, 555, 1233
2. Find GCF :

   $a.\ GCF(8,48);$ $b.\ GCF(7,15);$ $c. GCF(20,1000);$
   $d.\ GCF(23,69);$ $e.\ GCF(380,381);$ $f. GCF(14,25);$

3. Find GCF using prime decomposition:

   $a.\ GCF(75,135);$ $b.\ GCF(180,210);$ $c. GCF(125,462);$
   $d.\ GCF(504,270);$ $e.\ GCF(117,195,312);$ $f. GCF(306,340,850);$

4. A teacher divided 87 notebooks between the students in the class equally. How many students in the class and hoe many notebooks did each student get?

5. Mary wrote down a sequence of multiples of a certain number, starting with the smallest one. Twelfth number in this sequence is 60. Find the first, sixth, and twentieth numbers?

6. How many multiples of 9 among first 100 (natural) numbers?

7. Find the LCM using the prime decomposition:

   $a.\ LCM(28,35);$ $b.\ LCM(16,56);$ $c. LCM(21,100);$
   $d.\ LCM(18,62);$ $e.\ LCM(264,300);$ $f. LCM(360,1020);$
   $g.\ LCM(72,90,96);$ $h.\ LCM(58,87,435);$

8. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest number of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

9. There are less than 100 apples in a box. They can be evenly divided between 2, 3, 4, 5, and 6 kids. How many apples are there in the box?

10. Fill in the table. Find a pattern. What can you say about GCF, LCM and a product of two numbers

Numbers	Product	GCD(GCF)	LCM
4 and 6	24	2	12
6 and 9			
5 and 7			
35 and 45			
16 and 18			
735 and 845	$735 \cdot 845$		

Can you explain what you noticed?

11. Two buses leave from the same bus station following two different routes. For the first bus, it takes 48 minutes to complete the round trip route. For the second one it takes 1 hour and 12 minutes to complete the round-trip route. How much time will it take for the buses to meet at the bus station for the first time after they have departed for their routes at the same time?

12. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different kinds of flowers in each flower bed. What is the largest possible size of her square flower bed?