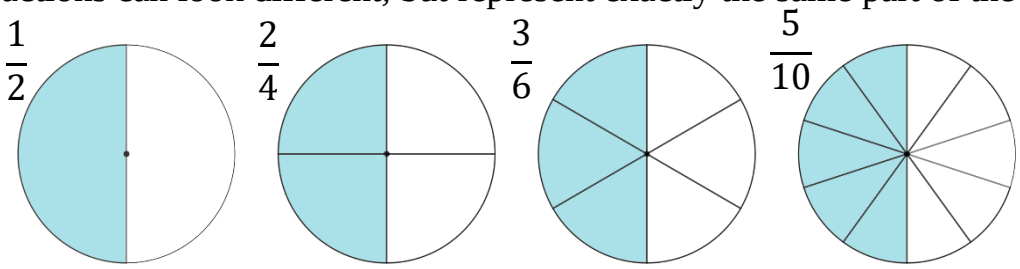
Math 4a. Class work 6. **Fractions. Addition, subtraction, multiplication, division.**

Equivalent fractions.

Some fractions can look different, but represent exactly the same part of the whole.



$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{5}{10};\qquad \frac{1}{2}=\frac{1\cdot 2}{2\cdot 2}=\frac{1\cdot 3}{2\cdot 3}=\frac{1\cdot 5}{2\cdot 5}$$

We can multiply the numerator and denominator of a fraction by the same number (not equal to 0), fraction will not change, it's still the same part of the whole. We're only dividing the whole into smaller parts and taking more such parts: if parts are twice smaller (denominator is multiplied by 2), we need twice more such parts to keep the fraction the same (numerator is multiplied by 2).

Fill the empty spaces for fractions:

$$\frac{2}{3}=\frac{}{9}=\frac{}{21}=\frac{4}{}=\frac{36}{}$$

This property of fractions can be used to reduce fractions. If there are common factors in the numerator and denominator, both numbers can be divided by common factors.

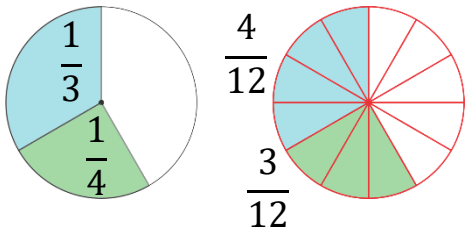
$$\frac{25}{35}=\frac{5\cdot 5}{7\cdot 5}=\frac{5}{7};\qquad \frac{77}{352}=\frac{7\cdot 11}{32\cdot 11}=\frac{7}{32}$$

Addition of the fraction with the same denominator is easy,

$$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$$

We divided a whole into 7 equal parts, took 2 of the small parts and then took 3. The result is 5 of $\frac{1}{7}$ parts of a whole. If denominators are different, it's not so easy anymore.

For example, adding $\frac{1}{3}$ and $\frac{1}{4}$: what part of the whole is the result? To figure it out, we need to find a number of small, equal parts into which the whole can be divided so that it is the common multiple for both denominators.



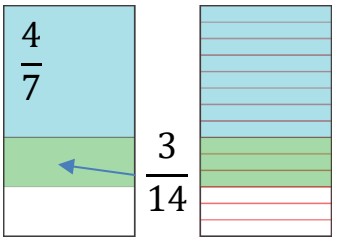
For 3 and 4 this number is 12, there are no common factors for 3 and 4, so we just need to multiply them.
$12:3=4, 12:4=3$

3 smaller $\frac{1}{12}$ parts can fit to $\frac{1}{4}$ and 4 smaller $\frac{1}{12}$ parts can fit to $\frac{1}{3}$.

$$\frac{1}{3}+\frac{1}{4}=\frac{1\cdot 4}{3\cdot 4}+\frac{1\cdot 3}{4\cdot 3}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$$

Another example:

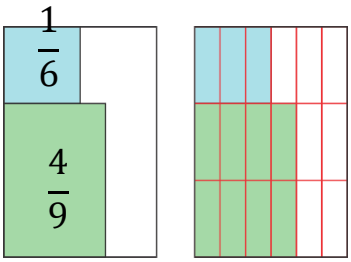
$$\frac{4}{7}+\frac{3}{14}=\frac{4\cdot 2}{7\cdot 2}+\frac{3}{14}=\frac{8}{14}+\frac{3}{14}=\frac{11}{14}$$



The common denominator for these two fractions is 14, least common multiple for 7 and 14.

One more example:

$$\frac{1}{6}+\frac{4}{9}=\frac{1\cdot 3}{6\cdot 3}+\frac{4\cdot 2}{9\cdot 2}=\frac{3}{18}+\frac{8}{18}=\frac{11}{18}$$



The most convenient common denominator is 18, LCM of 6 and 9. 54 (the product) also can be the common denominator, but the calculations will more complicated and the final fraction will need to be reduced:

$$\frac{1}{6}+\frac{4}{9}=\frac{1\cdot 9}{6\cdot 9}+\frac{4\cdot 6}{9\cdot 6}=\frac{9}{54}+\frac{24}{54}=\frac{33}{54}=\frac{11\cdot 3}{18\cdot 3}=\frac{11}{18}$$

Multiplication of fraction by a number.

To multiply a fraction by a number we just need to multiply the numerator by the number:

$$\frac{2}{7}\cdot 3=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{2+2+2}{7}=\frac{3\cdot 2}{7}=\frac{6}{7}$$

On the other hand:

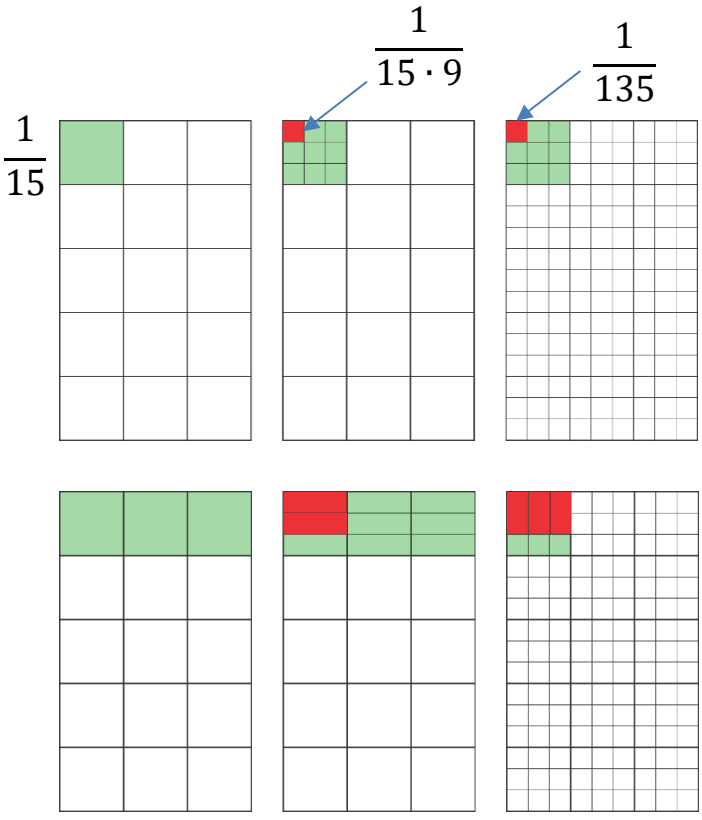
$$\frac{2}{7}\cdot 3=3\cdot\frac{2}{7}=3:7\cdot 2=3\cdot 2:7$$

Multiplication of fraction by a fraction.

$\frac{1}{15}$ is a part of a whole divided into 15 equal small parts.

If we want to take $\frac{1}{9}$ part of this little $\frac{1}{15}$ chunk we have to divide it into 9 even smaller pieces, to find $\frac{1}{9}^{\text{th}}$ of $\frac{1}{15}^{\text{th}}$.

$$\frac{1}{15}:9=\frac{1}{15}\cdot\frac{1}{9}=\frac{1}{15\cdot 9}=\frac{1}{135}$$



If we need to take two small $\frac{1}{9}$ of $\frac{1}{15}$

$$\frac{1}{15}:9\cdot 2=\frac{1}{15}\cdot\frac{2}{9}=\frac{1\cdot 2}{15\cdot 9}=\frac{2}{135}$$

Or we want to find out $\frac{2}{9}$ of $\frac{3}{15}$.

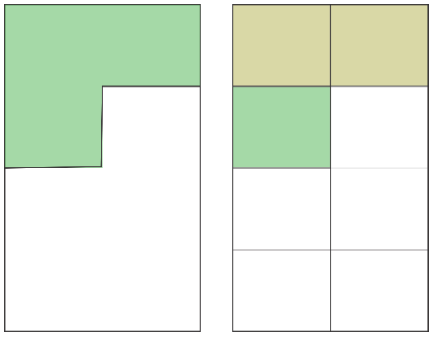
$$\frac{3}{15}:9\cdot 2=\frac{3}{15}\cdot\frac{2}{9}=\frac{3\cdot 2}{15\cdot 9}=\frac{6}{135}$$

To multiply two fractions, we need to multiply numerators, multiply denominators and reduce fraction, if possible.

Examples:

$$\frac{3}{8}\cdot\frac{2}{7}=\frac{3\cdot 2}{4\cdot 2\cdot 7}=\frac{3\cdot 2}{4\cdot 7\cdot 2}=\frac{3}{4\cdot 7}=\frac{3}{28}$$

Division of fractions.



More of multiplication of fractions:

$\frac{3}{8} \cdot \frac{2}{3} = \frac{2}{8} = \frac{1}{4}$

So, division of $\frac{1}{4}$ by $\frac{2}{3}$ should give the quotient $\frac{3}{8}$.

$\frac{1}{4} : \frac{2}{3} = \frac{3}{8}$

We can notice that the multiplication of $\frac{1}{4}$ by the inverse fraction $\frac{3}{2}$ will bring exactly $\frac{3}{8}$;

$$\frac{1}{4} : \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

To divide one fraction by another we need to multiply the dividend by the inverse fraction. Two fractions are inverse fractions if their product is 1. Inverse fractions can also be called reciprocal.

Examples:

$$\frac{1}{4} \cdot \frac{4}{1} = 1; \qquad \frac{3}{5} \cdot \frac{5}{3} = 1; \qquad \frac{4}{7} \cdot \frac{7}{4} = 1;$$

Exercise:

1. Bring the following fractions to denominator 36, if possible:

$$\frac{7}{12}; \quad \frac{7}{11}; \quad \frac{7}{10}; \quad \frac{7}{9}; \quad \frac{7}{8}; \quad \frac{7}{7};$$

2. Simplify the following fractions:

$$\frac{2 \cdot 3}{4 \cdot 5}; \quad \frac{2 \cdot 3}{7 \cdot 2}; \quad \frac{5 \cdot 4}{4 \cdot 9}; \quad \frac{7 \cdot 5}{2 \cdot 7}$$

$$\frac{22}{66}; \quad \frac{125}{75}; \quad \frac{75}{100}; \quad \frac{24}{360}; \quad \frac{125}{1000}; \quad \frac{100}{250}; \quad \frac{198}{126}$$

3. Painter painted $\frac{2}{7}$ of the house is 4 days. How many days will take him to paint the whole house?

4. Evaluate:

   $a.\ \frac{1}{2}-\frac{1}{4}+\frac{3}{5};$ $b.\ \frac{3}{4}-\frac{1}{2}+\frac{7}{8};$ $c.\ \frac{5}{6}-\frac{2}{3}+\frac{1}{4};$

5. Evaluate:

   $\frac{3}{7}\cdot 2;$ $3\cdot\frac{1}{6};$ $9\cdot\frac{5}{6};$ $2\frac{1}{3}\cdot 2;$ $4\cdot 1\frac{1}{2};$