

Math 4a. Class work 7. **Fractions. Addition, subtraction, multiplication, division.**

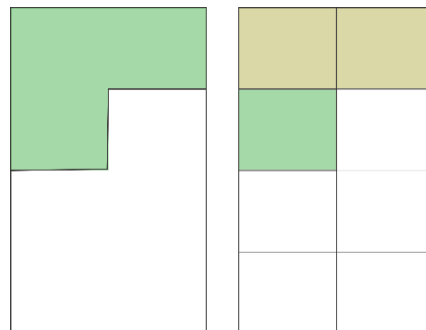


**Equivalent fractions.**

**Division of fractions.**

More of multiplication of fractions:

$$\frac{3}{8} \cdot \frac{2}{3} = \frac{2}{8} = \frac{1}{4}$$



We know that an operation of division is a process of finding of a number, which gives a dividend (the number we are dividing) if multiplied by a divisor.

For a natural number:

$$21:3 = 7, \quad 7 \cdot 3 = 21$$

Same should be applied to fractions too. If  $\frac{3}{8}$  multiplied by  $\frac{2}{3}$  is  $\frac{1}{4}$ ; then  $\frac{1}{4}$  divided by  $\frac{2}{3}$  should give quotient  $\frac{3}{8}$ .

$$\frac{1}{4} : \frac{2}{3} = \frac{3}{8}$$

We can notice that the multiplication of  $\frac{1}{4}$  by the inverse fraction  $\frac{3}{2}$  will bring exactly  $\frac{3}{8}$ ;

$$\frac{1}{4} : \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

To divide one fraction by another we need to multiply the dividend by the inverse fraction. Two fractions are inverse fractions if their product is 1. Inverse fractions can also be called reciprocal.

Examples:

$$\frac{1}{4} \cdot \frac{4}{1} = 1; \quad \frac{3}{5} \cdot \frac{5}{3} = 1; \quad \frac{4}{7} \cdot \frac{7}{4} = 1;$$

Let's solve a few problems with fractions:

1. Father is 42 years old. The son's age is  $\frac{2}{7}$  of his father. How old is the son?

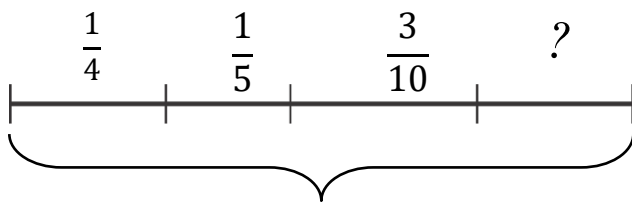
$42:7 = 6$  One seventh of the age of father is 6,  $6 \cdot 2 = 12$ , two sevenths is 12, son is 12 years old.

$$42:7 \cdot 2 = 42 \cdot \frac{1}{7} \cdot 2 = \frac{2}{7} \cdot 42 = 12$$

To find a part of a number, we need to multiply the part ( $\frac{3}{7}$ ) by a number (42).



2. A cyclist covered  $\frac{1}{4}$  of the distance in the first hour. During the second hour he drove  $\frac{1}{5}$  of the distance, and during the third hour he covered  $\frac{3}{10}$  of the distance. Which part of the planned distance does he still need to cover?



We need to add

$$\frac{1}{4} + \frac{1}{5} + \frac{3}{10};$$

The common denominator is 10,

$$\frac{1}{4} + \frac{1}{5} + \frac{3}{10} = \frac{1 \cdot 5}{4 \cdot 5} + \frac{1 \cdot 4}{4 \cdot 5} + \frac{3 \cdot 2}{10 \cdot 2} = \frac{5 + 4 + 6}{20} = \frac{15}{20} = \frac{3}{4}$$

$$1 - \frac{3}{4} = \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$$

3. Peter solved 12 math problems, and it's  $\frac{3}{5}$  of his assignment.

How many problems Peter need to solve to do his assignment?

If 12 is  $\frac{3}{5}$ ; one fifth will be  $12:3$ , and then it needs to be multiple 5, to find out how much the whole assignment is.

$$12:3 \cdot 5 = \frac{12}{3} \cdot 5 = 12 \cdot \frac{5}{3} \quad \text{or} \quad 12:\frac{3}{5};$$

To find a number by its known part, we need to divide the known part by a fraction representing this part.



**Addition, multiplication, and division of mixed numbers.**

To add two mixed numbers, we need to add the whole parts and the fractional parts, if the addition of fractional parts produces an improper fraction, the whole part of it should be added to rest whole parts.

$$\begin{aligned} 5\frac{9}{10} + 2\frac{4}{5} &= 5 + \frac{9}{10} + 2 + \frac{4}{5} = 5 + 2 + \frac{9}{10} + \frac{4}{5} = 7 + \frac{9}{10} + \frac{4 \cdot 2}{5 \cdot 2} = 7 + \frac{9+8}{10} = 7 + \frac{18}{10} \\ &= 7 + \frac{9 \cdot 2}{5 \cdot 2} = 7 + \frac{9}{5} = 7 + \frac{5}{5} + \frac{4}{5} = 7 + 1 + \frac{4}{5} = 6\frac{4}{5} \end{aligned}$$

To multiply or divide mixed numbers, we need to represent them as improper fractions:

$$1\frac{1}{7} \cdot 3\frac{1}{16} = \left(\frac{7}{7} + \frac{1}{7}\right) \cdot \left(\frac{3 \cdot 16}{16} + \frac{1}{16}\right) = \frac{8}{7} \cdot \frac{49}{16} = \frac{8 \cdot 7 \cdot 7}{7 \cdot 8 \cdot 2} = \frac{7}{2} = 3\frac{1}{2}$$

$$10\frac{3}{5} : 1\frac{23}{30} = \left(10 + \frac{3}{5}\right) : \left(1 + \frac{23}{30}\right) = \left(\frac{50}{5} + \frac{3}{5}\right) : \left(\frac{30}{30} + \frac{23}{30}\right) = \frac{53}{5} : \frac{53}{30} = \frac{53}{5} \cdot \frac{30}{53} = \frac{30}{5} = 6$$

Exercises:

1. Evaluate:

$$a. \frac{1}{3} \cdot \frac{2}{7}; \quad b. \frac{1}{2} \cdot \frac{5}{6}; \quad c. \frac{3}{4} \cdot \frac{1}{4}; \quad d. \frac{1}{2} \cdot \frac{1}{3}$$

$$e. \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{4}{9}; \quad f. \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}; \quad g. \frac{4}{7} \cdot \frac{35}{36} \cdot \frac{3}{5};$$

$$h. \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{23}{24} \cdot \frac{24}{25}$$

2. Evaluate:

$$a. \frac{14}{15} \cdot \frac{10}{49} + 5\frac{3}{7}; \quad b. 1\frac{3}{11} - \frac{27}{44} \cdot \frac{4}{9}; \quad c. 1\frac{2}{3} + \frac{14}{15} \cdot \frac{5}{7}$$

3. Evaluate:

$$a. \frac{3}{3} : \frac{5}{7}; \quad b. \frac{1}{4} : \frac{1}{2}; \quad c. \frac{3}{4} : \frac{1}{2}; \quad d. \frac{4}{9} : \frac{8}{9}$$

e.  $2:\frac{1}{7}$ ;      f.  $4:\frac{3}{5}$ ;      g.  $\frac{10}{21}:5$ ;      h.  $\frac{2}{3}:4$

i.  $3\frac{1}{2}:2\frac{1}{3}$ ;      j.  $10\frac{1}{2}:3\frac{1}{2}$ ;      k.  $5\frac{1}{2}:3\frac{2}{3}$ ;      l.  $3\frac{3}{5}:6\frac{6}{4}$

4. A 5 kg cake can be divided into how many equal  $\frac{1}{5}$  kilogram portions?
5. If only  $\frac{3}{10}$  of a kilogram of gem fits into a jar, how many jars are needed to hold 2 kilograms of gem
6. Evaluate:  
a.  $14:42$ ;    b.  $2:3:5$ ;    c.  $2:8 \cdot 3$ ;    d.  $100 \cdot 6:40$ ;    e.  $5:15 \cdot 3$
7. To do his homework, Peter needs to write an essay and solve math problems. He spent  $\frac{1}{2}$  hours doing his homework. However, the time he spent writing an essay was  $\frac{1}{10}$  hours less than the time he spent solving math problems. How much time did he dedicate to working on math problems?
8. Julia spent  $\frac{1}{3}$  of her money and then \$10 more. She spent a half of her money in total. How much money did she have?
9. There are 24 pages in my notebook.  $\frac{3}{8}$  of all pages have been used. How many pages are still blank?