Math 4a. Class work 8.

Complex fractions.

Complex fractions are formed by two fractional and/or numeral expressions, one on the top and the other one on the bottom, for example:

$$\frac{(2+3)\cdot 5}{7-\frac{1}{2}}; \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{7}{9}-\frac{2}{5}}$$

We know that the fraction bar is just another way to write the division sign, so the above expressions are equivalent to

$$\frac{(2+3)\cdot 5}{7-\frac{1}{2}} = ((2+3)\cdot 5):\left(7-\frac{1}{2}\right); \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{1}{4}} = (\frac{1}{2}+\frac{1}{3}) \div (\frac{2}{3}+\frac{1}{4})$$

It is easy to simplify a complex fraction:

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{1}{4}} = \left(\frac{1}{2}+\frac{1}{3}\right) \div \left(\frac{2}{3}+\frac{1}{4}\right) = \frac{\frac{3}{6}+\frac{2}{6}}{\frac{8}{12}+\frac{3}{12}} = \frac{\frac{5}{6}}{\frac{11}{12}} = \frac{5}{6} \div \frac{11}{12} = \frac{5}{6}\cdot\frac{12}{11} = \frac{5}{1}\cdot\frac{2}{11} = \frac{10}{11}$$

Problem solving examples:

1. *What number x can be substituted with so that the fraction $\frac{x}{18}$ will be a nonreducible proper fraction?*

For the fraction $\frac{x}{18}$ to be proper x should be less than 18, and greater than 0. We have numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17. All of them greater than 0 and less than 18. For the fraction to be nonreducible, the numerator and denominator should not have common factors. 18 can be prime factorized as:

$$18 = 2\cdot 3\cdot 3$$

So, we have to exclude all even numbers, 2, 4, 6, 8, 10, 12, 14, 16.
The numbers left are 1, 3, 5, 7, 9, 11, 13, 15, 17.
Then we have to exclude all numbers divisible by 3: 3, 9, 15. Numbers that are not divisible by 3 will not be divisible by 6 $(2\cdot 3)$ and 9 $(3\cdot 3)$ as well. So, x can be substituted with 1, 5, 7, 11, 13, and 17. All these fractions are proper and nonreducible:

$$\frac{1}{18}; \frac{5}{18}; \frac{7}{18}; \frac{11}{18}; \frac{13}{18}; \frac{17}{18};$$

2. *56 tickets were sold for the flight, and 24 seats remained unoccupied. What fraction of the seats are occupied?*

Step 1. Total number of seats is $56 + 24 = 80$

Step 2. One seat is $\frac{1}{80}$ part of the total number of seats. 56 such parts are $\frac{56}{80}$.

$$56:80 = \frac{56}{80} = \frac{7 \cdot 8}{10 \cdot 8} = \frac{7}{10}$$

Answer: $\frac{7}{10}$ of all seats are occupied.

Exercises:

1. Evaluate:

$a.\ \dfrac{6}{1-\frac{1}{3}}$; $b.\ \dfrac{1-\frac{1}{6}}{2+\frac{1}{6}}$; $c.\ \dfrac{\frac{1}{2}+\frac{3}{4}}{\frac{1}{2}}$; $d.\ \dfrac{\frac{7}{10}+\frac{1}{3}}{\frac{7}{10}+\frac{1}{2}}$; $e.\ \dfrac{2-\frac{\frac{1}{2}-\frac{1}{4}}{2}}{2+\frac{\frac{1}{2}-\frac{1}{4}}{2}}$

2. Write the expressions as fractions and evaluate:

$a.\ 14:42$; $b.\ 2:3:5$; $c.\ 2:8 \cdot 3$; $d.\ 100 \cdot 6:40$; $e.\ 5:15 \cdot 3$

$f.\ (21 \cdot 18):14$; $g.\ 50:(16 \cdot 25)$; $h.\ (12 \cdot 15):40$; $i.\ (4 \cdot 24):(2 \cdot 8)$

3. 5 kg cake can be divided into how many equal $\frac{1}{5}$ kilogram portions?

4. To do his homework, Peter needs to write an essay and solve math problems. He spent $\frac{1}{2}$ hours doing his homework. However, the time he spent writing an essay was $\frac{1}{10}$ hours less than the time he spent solving math problems. How much time did he dedicate to working on math problems?

5. Julia and Mary ate all the Halloween candy. Mary claimed that she ate $\frac{2}{3}$ of the candies, and Julia said that she ate $\frac{3}{5}$. Their parents think that something is wrong. Are they right?

6. The model of the house is $\frac{1}{25}$ of its real size. The width of a window on the model is 5 cm. How wide is a window in a real house?

7. What is the length of a segment if
   a. $\frac{2}{5}$ of its length is 12 meters;
   b. $\frac{3}{4}$ of its length is 9 centimeters;
   c. $\frac{3}{5}$ of its length is 15 millimeters.
   d. $\frac{2}{7}$ of its length is 8 meters.

8. What number x can be substituted with so that the fraction $\frac{x}{12}$ will be a nonreducible propre fraction?

9. From 42 m of fabric, 10 identical duvet covers were sewn, and from 33 m - 15 identical sheets. How much fabric is needed for a set that includes 1 sheet and 1 duvet cover?

10. Mary’s 10 steps are 9 meters, while Julia's 20 steps are 17 meters. Who’s step is longer?

11. The sum of all numbers in each square is 10. What number should be placed instead of “?” ?

$2\frac{1}{7}$	$5\frac{4}{7}$
$\frac{3}{7}$	?

$1\frac{4}{5}$	$3\frac{2}{5}$
?	$2\frac{1}{5}$

$\frac{5}{9}$	?
$2\frac{7}{9}$	$1\frac{2}{9}$

?	$6\frac{8}{11}$
$\frac{2}{11}$	$2\frac{5}{11}$