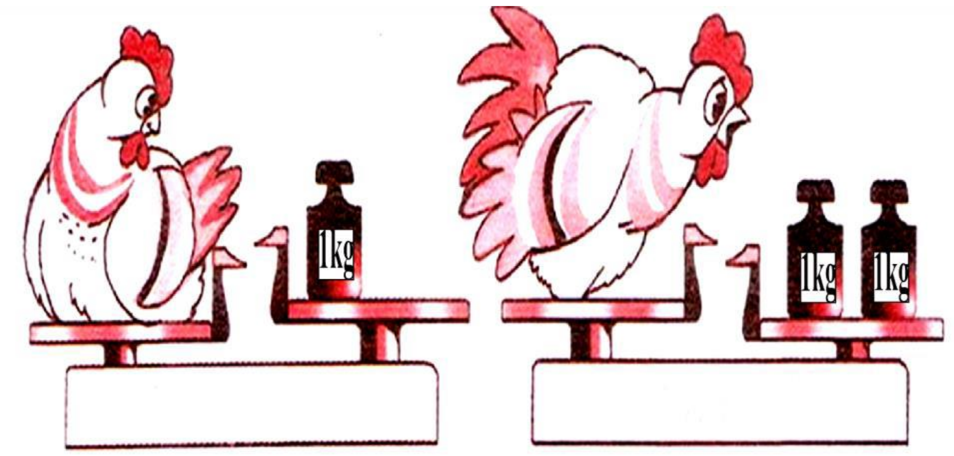
Math 4a. Classwork 13.

Decimals

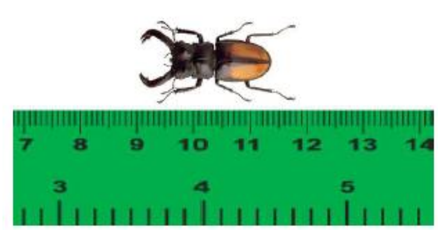
In a process of measurement, we compare a standard unit, such as 1m for length, 1kg for mass, 1degree Celsius for temperature, and so on (we can use another standard units, for example 1 foot, 1 degree Fahrenheit) with the quantity we are measuring. It is very likely that our measurement will not be exact and whole



number of standard units will be either smaller, or greater than the measured quantity. In order to carry out more accurate measurement we have to break our standard unit into smaller equal parts. We can do this in many different ways. For example, we can take $\frac{1}{2}$ of a standard unit and continue measuring. If we didn't get exact n units plus $\frac{1}{2}$ of a unit we have to subdivide further:

$$n + \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{1}{2}\right) + \cdots$$

It turns out that perhaps the most convenient way is to divide a unit into 10 equal parts, then each of one tenth into another 10 even smaller equal parts and so on. In this way we will get a series of fractions with denominators 10, 100, 1000 and so on:

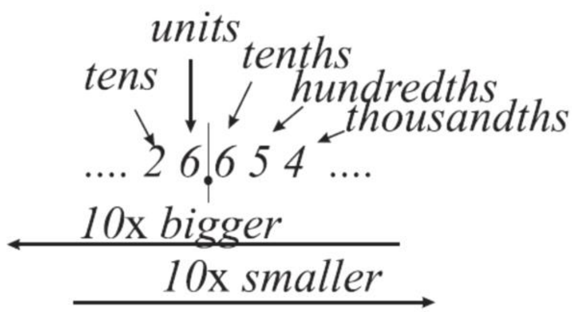


$$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000} \ldots.$$

The result of our measurement can be written in a 10 based place value system.

$$26.654 = 10\cdot 2 + 1\cdot 6 + \frac{1}{10}\cdot 6 + \frac{1}{100}\cdot 5 + \frac{1}{1000}\cdot 4 = 10\cdot 2 + 1\cdot 6 + \frac{6}{10}\cdot + \frac{5}{100} + \frac{4}{1000}$$
$$= 10\cdot 2 + 1\cdot 6 + \frac{600}{1000} + \frac{50}{1000} + \frac{4}{1000}$$

Of course, all such numbers can be expressed in the fractional notation as fractions with denominators 10, 100, 1000 …, but in decimal notation all arithmetic operations are much easier to perform.



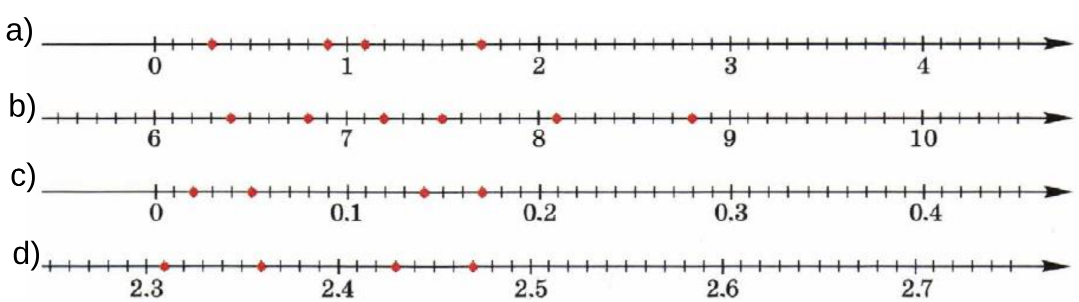
Exercises:

1. Write in decimal notation the following fractions:
   Example:

$$1\frac{3}{25} = 1 + \frac{3}{25} = 1 + \frac{3\cdot 4}{25\cdot 4} = 1 + \frac{12}{100} = 1.12$$

$1\frac{1}{10}$; $2\frac{4}{10}$; $4\frac{9}{10}$; $24\frac{25}{100}$; $98\frac{3}{100}$; $1\frac{1}{100}$; $4\frac{333}{1000}$; $8\frac{45}{1000}$; $75\frac{8}{10000}$; $9\frac{565}{10000}$

2. Which numbers are marked on the number lines below:



3. Evaluate:

$a.\ 1.2 + 2.3 + 3.4 + 4.5 + 5.6 + 6.7 + 7.8;$

$b.\ 2.3 + 3.4 + 4.5 - 5.6 + 6.7 + 7.8 + 8.5 + 9.2;$

$c.\ 1.7 + 3.3 + 7.72 + 3.28 + 1.11 + 8.89;$

$d.\ 18.8 + 19 + 12.2 + 11.4 + 0.6 + 11;$

4. On a graph paper draw a number line, use 10 squares as a unit. Mark points with coordinates 0.1, 0.5, 0.7, 1.2, 1.3, 1.9.

5. Which fractions below can be written in as a finite decimal:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}.$$

Why do you think so?

6. Write decimals as fractions and evaluate the following expressions:

$a.\ \frac{2}{3} + 0.5;$ $b.\ \frac{1}{3} \cdot 0.9;$ $c.\ \frac{3}{16} \cdot 0.16$

$d.\ 0.6 - \frac{2;}{5}$ $e.\ 0.4 : \frac{2}{7};$ $f.\ \frac{9}{20} : 0.03$

7. Which part of 1 m is 1 cm?
Which part of 1 km is 1 m?
Which part of 1 cm is 1 mm?
Which part of 1 m is 1 dm?
Which part of 1 kg is 1 g?
Which part of 1 g is 1 mg?

8. 1 kilogram of candies costs 16 dollars. How much
   a. 0.5 kg will cost?
   b. 1.2 kg will cost?
   c. 0.75 kg will cost?
   d. 0.4 kg will cost?
   e. 2.5 kg will cost?