Math 4a. Classwork 17.

About variables.

When we need wright the mathematical expression, but we don't know exact numbers to be used, we use variables. It can be any symbol, but it's very convenient to use letters. For example, if the number of the books on one shelf is n and the number of the book on the other shelf is m, the total number of books on both shelves is $n + m$. We can perform all the usual arithmetic operation with the variables, but the exact answer can be reached only when the values are passed to the variables.

Write the expression for the following problems:

3 packages of cookies cost a dollars. How many dollars do 5 of the same packages cost?

If 3 packages of cookies cost a dollars, the price of one pack is

$$1pack = \frac{a}{3} = a:3$$

Five such packs will cost

$$5 \cdot a:3 = \frac{5a}{3} = \frac{5}{3}a$$

5 bottles of juice cost b dollars. How many bottles can one buy with c dollars?

Again, if 5 bottles cost b dollars, the price of one bottle is

$$\frac{b}{5} \text{ dollars}$$

If I have only c dollars, I can buy the number of bottles equal to my total money divided by the price of one bottle:

$$c:\frac{b}{5} = c \cdot \frac{5}{b} = \frac{5c}{b}$$

If I have only \$30 and 5 bottles cost 10 dollars I can buy:

$$30:\frac{10}{5} = 30 \cdot \frac{5}{10} = 30 \cdot \frac{1}{2} = 15 \text{ bottles}$$

Equation.

Equation is an equality with one or more variables. For example.

$1 + x = 15$

$x + y + z = 100$ $\quad\quad\quad\quad$ $|x| = 10$

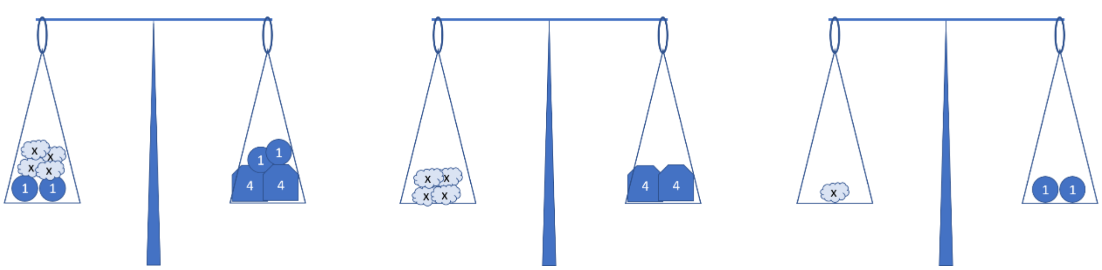
$z \cdot y = 1$ $\quad\quad\quad\quad$ $|x| = -10$

To solve an equation means to find such value of the variable(s) that the equation will become a true equality.
In the first equation above, if the value of *x* is 14, the right side of the equality is equal to the left side:
$1 + 14 = 15$. Equation $|x| = -10$ doesn't have any solution for *x*, because there is no number with absolute value which is less than 0.
Another examples of equations:

$$5y = -100$$
$$4 + 3x = 25$$
$$34 = 2x + 4$$
$$2 + 4x = 10$$

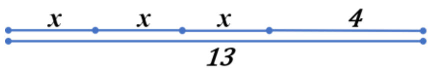
What can we do with an equation to solve it? As an example, we can solve the equation $2 + 4x = 10$. We can add (and subtract) the same quantity to (from) both sides of an equation, the balance will be in place. Also, we can multiply and divide both side by the same amount, kipping the balance.

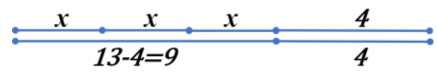
$2 + 4x = 10$ $\quad\quad$ $2 - 2 + 4x = 10 - 2$ $\quad\quad$ $(4:4)x = 8:4$



The process of the solving equation can be visualized in a different way. Let solve another equation:

$3x + 4 = 13$
$3x = 13 - 4 = 9$
$x = 9 : 3 = 3$





Substitution.

Let's take a look at a very simple equation.

$$|x| = 10$$

The solution to this equation is a number, the absolute value of which is 10. There are two such numbers, 10 and (-10). Thus, this equation has two roots. (The word "root" can be used as a synonym for solution.).

$$|x| + 5 = 10$$

To make the equation a little simpler, we can substitute $|x|$ with m ($|x| = m$) and solve for m.

$$m + 5 = 10$$

$$m + 5 - 5 = 10 - 5$$

$$m = 5.$$

But the initial variable is x, not m. $|x| = m$, or, as we know, $|x| = 5$. There are two roots, 5 and (-5).

Equations are very useful to solve word problems. In each word problem there is an unknown quantity, and known parameters out of which the equation can be created. For example, let's take a look on the following problem:

There are 27 pencils in two boxes altogether. There are 5 more pencils In one box then in the other. How many pencils are there in each box?

There are two unknown quantities in this problem, the number of pencils in the first box and the number of pencils in the second box. But these two quantities are not independent, one is 5 less than the other. If the number of pencils in one box is denoted as x, number of pencils in the second box will be $x + 5$. And we also know that the total number is 27.

$$x + x + 5 = 27$$

$$2x = 27 - 5 = 22$$

$$x = 22:2 = 11$$

Answer: there are 11 pencils in one box, and 16 in the other.

There are candies in box. If each kid will take 4 candies, 19 candies will be left in the box. If each kid will take 5 candies, there will be lacking 2 candies. How many candies are there in the box?

In this problem there are also two unknown quantities, the number of kids, and number of candies in the box. If the number of kids is denoted as x, the number of candies can be calculated in to ways:

First, $5 \cdot x - 2 =$ number of candies in the box

Second, $4 \cdot x + 19 =$ number of candies in the box, so

$$5 \cdot x - 2 = 4 \cdot x + 19$$

$$5x - 4x = 19 + 2$$

$$x = 21$$

The number of kids is 21. The number of candies can be calculated from either expression:

$5 \cdot 21 - 2 = 4 \cdot 21 + 19 = 103$

Answer: there are 103 candies in the box.

There were 624 books in two boxes altogether. When $\frac{1}{3}$ *of the books from one box and* $\frac{3}{7}$ *of the books from another box were sold to the customers, the number of books in each box became equal. How many books there were in each box at the beginning?*

In this problem there are two unknown variables, number of books in each box. Let's denote the number of books in the first box as x, and the number of books in the second box as y. Together $x + y = 624$. But we know also that

$$\frac{2}{3}x = \frac{4}{7}y$$

$$x = \frac{4}{7}y \cdot \frac{3}{2} = \frac{4 \cdot 3}{7 \cdot 2}y = \frac{6}{7}y$$

We can now substitute x in the equation $x + y = 624$ with $\frac{6}{7}y$.

$$\frac{6}{7}y + y = 624$$

$$\frac{13}{7}y = 624$$

$$y = 624 \cdot \frac{7}{13} = 48 \cdot 7 = 336$$

$$x = \frac{6}{7} \cdot 336 = 288$$

Answer: 288 books, and 336 books.

On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?

Again, there are two unknown amounts in the problem: number of yellow and number of white dandelions at the beginning, the sum of these two numbers is 35. We can use y and w as variable names for convenience.

$$y + w = 35$$

Which gives us the following relationship:

$$w = 35 - y$$

Also, we know that

$$2 \cdot (w - 8 + 2) = y - 2$$

$$2(w-6)=y-2$$

(eight whites are gone and two yellows are now white, and number of yellows now twice as big as number of whites). Using the substitution $w=35-y$, the last equation can be rewritten as

$$2(35-y-6)=y-2$$

$$2(29-y)=y-2$$

$$58-2y=y-2$$

$$58+2=y+2y$$

$$3y=60$$

$$y=20,\quad w=35-20=15$$

Answer: at the beginning there were 15 white and 20 yellow dandelions.

Do you have any idea how to solve this problem without writing un equations?

Exercises:

1. An apple costs x dollars and a pear costs y dollars. Explain the expressions below:

$$x+y,\qquad x-y,\qquad 3x,\qquad 8y,\qquad 3x+8y,\qquad y:x,\qquad 120:y$$

2. Write the following as mathematical expression. If this expression is an equation, solve it.
    a. Sum of the number x and 15 equals to 20.
    b. Product of y and10.
    c. Difference between three times z and 4 is equal to12.
    d. Half of the number b is equal to 1.5
    e. Product of the numbers of 5 and x is less than 12.
3. 10 identical notebooks cost x dollars. Textbook costs 15 dollars more than notebook.
    a. What is the price of one notebook?

b. What is the price of the textbook?

c. What is the price of n notebooks?

d. What is the price of n notebooks and m textbooks?

4. The sum of three consecutive odd numbers is 135. What is the smallest of the three numbers?

5. Mary bought 5 apples and 2 pears for $4.60. Eva bought 8 apples and 6 pears for $6.24. Veronica bought 3 apples and 3 pears. How much change did she get back from $5.00?

6. Solve the following equations:

$a.\ \ 2x + 3 = 11;$ $b.\ \ \frac{1}{2}x - 5 = 12;$ $c.\ \ 14 + x = 4 + 6x$