Math 4a. Classwork 19.

There are candies in box. If each kid will take 4 candies, 19 candies will be left in the box. If each kid will take 5 candies, there will be lacking 2 candies. How many candies are there in the box?

In this problem there are two unknown quantities, the number of kids, and number of candies in the box. If the number of kids is denoted as n, the number of candies can be calculated in to ways:

First, $5 \cdot n - 2 =$ number of candies in the box

Second, $4 \cdot n + 19 =$ number of candies in the box, so

$$5 \cdot n - 2 = 4 \cdot n + 19$$

$$5n - 4n = 19 + 2$$

$$n = 21$$

The number of kids is 21. The number of candies can be calculated from either expression:

$5 \cdot 21 - 2 = 4 \cdot 21 + 19 = 103$

Answer: there are 103 candies in the box.

Can this problem be solved without writing the equations? Number of candies in a box divided by a number of kids is giving us the quotient 4 and remainder 19. If we try to divide in a way to get a quotient 5, the 'negative' remainder 2 (lacking) will occur, so $19 + 2$ is a divisor, number of kids at the party.

There were 624 books in two boxes altogether. When $\frac{1}{3}$ of the books from one box and $\frac{3}{7}$ of the books from another box were sold to the customers, the number of books in each box became equal. How many books were there in each box at the beginning?

In this problem there are two unknown variables, number of books in each box. Let's denote the number of books in the first box as x, and the number of books in the second box as y. Together $x + y = 624$. But we also know that

$$\frac{2}{3}x = \frac{4}{7}y$$

$$x = \frac{4}{7}y : \frac{2}{3} = \frac{4}{7}y \cdot \frac{3}{2} = \frac{4 \cdot 3}{7 \cdot 2}y = \frac{6}{7}y$$

We can now substitute x in the equation $x + y = 624$ with $\frac{6}{7}y$.

$$\frac{6}{7}y + y = 624$$

$$\frac{13}{7}y = 624$$

$$y = 624 \cdot \frac{7}{13} = 48 \cdot 7 = 336$$

$$x = \frac{6}{7} \cdot 336 = 288$$

Answer: 288 books, and 336 books.

How this problem can be solved without writing the equation? $\frac{2}{3}$ of the number of books in the first box is equal to $\frac{4}{7}$ of number of books in the second box, so number of books in first box is $\frac{4}{7} + \frac{2}{7} = \frac{6}{7}$ of the number of books in the second box.

Totally in both boxes are $\frac{6}{7} + \frac{7}{7} = \frac{13}{7}$ of number of books in the second box and it's 624 books.

$624 : 13 \cdot 7 = 336$ books in the second box.

$624 - 336 = 288$ books in the first box.

On the lawn grew 35 yellow and white dandelions. After eight whites flew away, and two yellows turned white, there were twice as many yellow dandelions as white ones. How many whites and how many yellow dandelions grew on the lawn at the beginning?

Again, there are two unknowns in the problem: number of yellow and number of white dandelions at the beginning, the sum of these two numbers is 35. We can use y and w as variable names for convenience.

$$y + w = 35$$

Which gives us the following relationship:

$$w = 35 - y$$

Also, we know that

$$2 \cdot (w - 8 + 2) = y - 2$$

$$2(w - 6) = y - 2$$

(Eight whites are gone and two yellows are now white, and number of yellows now twice as big as number of whites). Using the substitution $w = 35 - y$, the last equation can be rewritten as

$$2(35 - y - 6) = y - 2$$

$$2(29 - y) = y - 2$$

$$58 - 2y = y - 2$$

$$58 + 2 = y + 2y$$

$$3y = 60$$

$$y = 20, \quad w = 35 - 20 = 15$$

Answer: at the beginning there were 15 white and 20 yellow dandelions.

Do you have any idea how to solve this problem without writing un equations?

Exercises:

1. Mouse and bird (toys) together cost 10 dollars. 5 mice and 6 birds cost 56 dollars. How much does a mouse and a bird cost separately? Write an equation and solve the problem. Solve the problem without formally writing out the equation.
2. Ten mice and birds (real, not toy) ate 56 grains. Each mouse ate 5 grains, and each bird ate 6 grains. How many mice and how many birds were there? Write an equation and solve the problem. Solve the problem without formally writing out the equation.

3. When one nightingale perched on each tree, one nightingale was left without a tree. And when nightingales sat on the trees in pairs, then one tree was left without nightingales. How many trees and how many nightingales were there?

4. Mary bought 5 apples and 2 pears for $4.60. Eva bought 8 apples and 6 pears for $6.24. Veronica bought 3 apples and 3 pears. How much change did she get back from $5.00?