Math 4b. Classwork 5.

**Problem1.** For Halloween the Jonson family bought 24 mini chocolate bars and 36 gummy worms. What is the largest number of kids between whom the Jonson can divide both kinds of candy evenly?

To solve this problem, we have to find a number which can serve as a divisor for 24 as well as for 36. There are several such numbers. The first one is 2. Both piles of candy can be evenly divided between just 2 kids. 3 is also a divisor, 4 is also a divisor, 6 is also a divisor and so on.

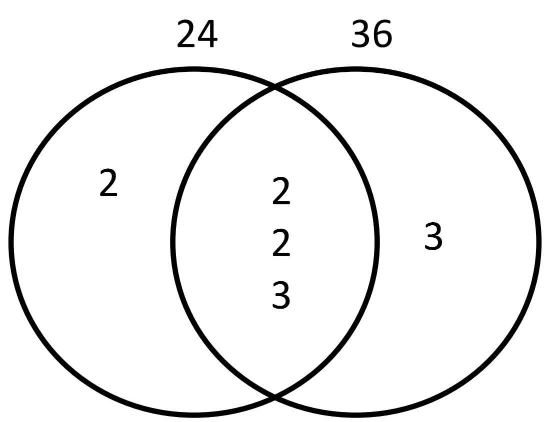
The Jonson family wants to treat as many kids as possible with equal numbers of candy. To do this they have to find **the Greatest Common Factor (GCF), the largest number that can be a divisor for both (24 and 36) amounts of candy.**

Let's take a look at all prime factors of 24 and 36. Prime factorization of 24 gives us 2, 2, 2, 3. Any of these numbers, as well as any of their products can be a divisor for 24. Prime factorization of 36 gives us 2, 2, 3, 3.

$$2 \cdot 2 \cdot 2 \cdot 3 = 24$$

$$2 \cdot 2 \cdot 3 \cdot 3 = 36$$

We see that these two representations have common factors 2, 2, 3.



It means that both numbers are divisible by any of these common factors and by any of their products. The largest product is the product of all common factors. This largest product has a name: **Greatest Common Factor,** or GCF. This GCF will also be a Greatest Common Divisor (GCD).

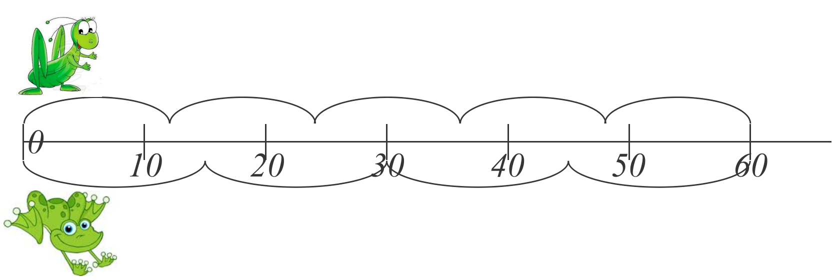
$$\mathrm{GCF}(24, 36) = 12$$

Between 12 kids they can divide both kinds of candy evenly.

**Problem 2.** A grasshopper jumps a distance of 12 centimeters each jump. A little frog leaps a distance of 15 centimeters each time. They both start at 0 and hop along the long ruler. What is the closest point on the ruler at which they can meet?

There are places on the ruler that both of them can reach after a certain number of jumps. One of such places is 12 x 15 cm. A grasshopper would make 15 jumps, while a frog would do only 12. Will be the only place where they can meet or are there other places?

Are there any other common multiples of 12 and 15, which are less then $12 \times 15$ and are still divisible by 12 and 15?




The number which we are looking for has to be a product of prime factors of 12 and 15.

Prime factorization of 12 and 15:

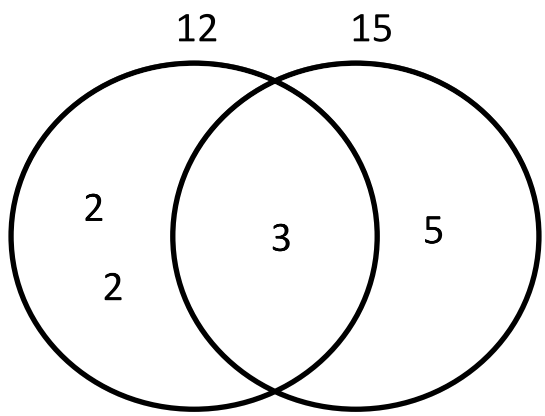
$$2 \cdot 2 \cdot 3 \quad = 12$$

$$3 \cdot 5 = 15$$

These factors should be used in sufficient, but least necessary quantities. We should use common factors only once.
The common factor of 12 and 15 is 3:

$$2 \cdot 2 \cdot 3 = 12$$

$$3 \cdot 5 = 15$$




So the smallest multiple is $2 \cdot 2 \cdot 3 \cdot 5 = 60$

60 is the smallest number, which is divisible by 12 and 15 and is called **Least common multiple (LCM).**

$$\mathrm{LCM}(12, 15) = 60$$

**Least common multiple of 2 numbers is the smallest number that is divisible by both numbers.**

**Problem 3.** The Johnson family wants to buy the same number of gammy worms and mini chocolates (168 mini per box and 180 gammy worms per box). How many boxes of each type of candy do they need to buy?

$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 168$$

$$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$$

$$7 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2520$$

If all factors from one number are multiplied by the factors from the second number, which are missing in the first one, the resulting number is divisible by both numbers (is a common multiple) and is the smallest common multiple.

Least common multiple for 168 and 180 is 2520.

They need to buy $2520 \div 168 = 15$ boxes of mini chocolates and

$2520 \div 180 = 14$ boxes of gammy worms.

**Homework**

Make sure to write your solutions and not just answers.

1. A florist has 36 roses and 60 daisies. What is largest number of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

2. Find GCF: $a.\ GCF(8,48)$ $b.\ GCF(7,15)$ $c. GCF(20,100)$

3. Find GCF using prime factorization:
   $a.\ GCF(75,125)$; $b.\ GCF(18,21)$; $c.\ GCF(48,40)$

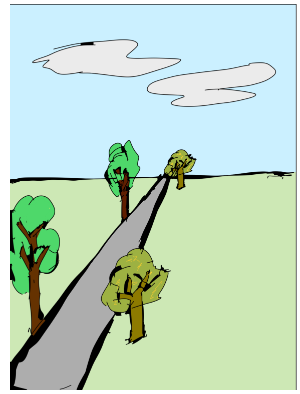
4. Mary has a rectangular backyard with sides of 48 and 40 yards. She wants to create square flower beds, all of equal size, and plant different kinds of flowers in each flower bed. What is the largest possible size of her square flower bed?

5. Find the LCM using the prime factorization:

   $a.\ LCM(12,9)$ $b. LCM(8,10)$ $c.\ LCM(18,72)$; $d. LCM(15,12,10)$

6. "Sweet Mathematics" sweets are sold in 12 pieces per box, and "Geometry with Nuts" sweets are sold in 15 pieces per box. What is the smallest number of boxes of both chocolates you need to buy to have equal number of both kind of chocolates?

7. Along the straight road, the landscaping company is planting maples on one side and oaks on the other. They planted an oak and a maple opposite each other at the start of the road, and then planted an oak every 16 feet and a maple every 18 feet. At what distance will oak and maple be planted opposite each other again?



8. Two buses leave from the same bus station following two different routes. For the first bus, it takes 48 minutes to complete the round-trip route. For the second one it takes 1 hour and 12 minutes to complete the round-trip route. How much time will it take for the buses to meet at the bus station for the first time after they have departed for their routes at the same time.
   (**Hint:** convert time from hours into minutes)

9. Fill in the table. Find a pattern. What can you say about GCF, LCM and a product of two numbers

| Numbers | Product | GCD(GCF) | LCM |
|---|---|---|---|
| 4 and 6 | 24 | 2 | 12 |
| 6 and 9 | | | |
| 5 and 7 | | | |
| 9 and 12 | | | |

Can you explaine what you noticed?