Math 4b. Classwork 9.

**About variables.**

When we need to write a mathematical expression, but we don't know the exact numbers to use, we use variables. It can be any symbol, for example ☆ or 😀, but it is very convenient to use letters. For example, if the number of books on the first shelf is $n$ and the number of books on the second shelf is $m$, the total number of books on both shelves is $n + m$. We can do all the usual arithmetic operations on variables, but the exact answer can only be obtained when values are passed into variables.

Let's have a look at expressions for the following problems:

- 3 packages of cookies cost $a$ dollars. How much do 5 such packages cost?

If 3 packages of cookies cost $a$ dollars, one pack costs

$$1pack = \frac{a}{3} = a:3$$

Five such packs will be

$$5 \cdot a:3 = \frac{5a}{3} = \frac{5}{3}a$$

- 5 bottles of juice cost $b$ dollars. How many bottles can one buy with $c$ dollars?

Similarly to the problem above, if 5 bottles cost b dollars, one bottle will cost

$$\frac{b}{5} \text{ dollars}$$

If I have only c dollars, I can buy the number of bottles equal to my total money divided by the price of one bottle:

$$c:\frac{b}{5} = c \cdot \frac{5}{b} = \frac{5c}{b}$$

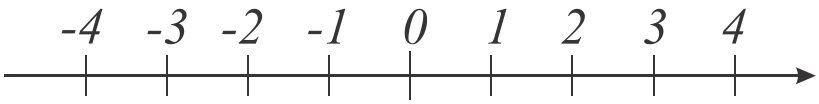
If I have only \$30 and 5 bottles cost 10 dollars I can buy:

$$30:\frac{10}{5} = 30 \cdot \frac{5}{10} = 30 \cdot \frac{1}{2} = 15 \text{ bottles}$$

**Positive and negative numbers.**

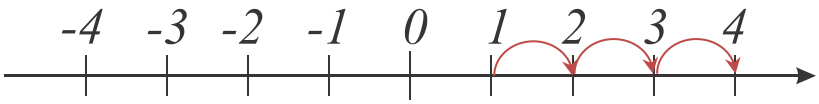
If positive represents above sea level, then negative represents below level. If positive represents a deposit, negative represents a withdrawal. If positive represents movement to the right, negative represents movement to the left.

Numbers to the left of zero on the number line are called **negative.** They are less than 0, and we write the "–" in front of them. The numbers to the right from zero are positive.



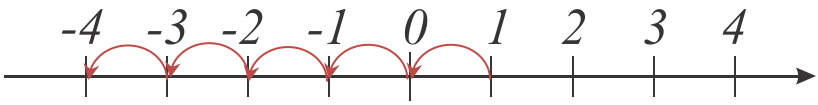

**Addition. Substruction.**
If we add a positive number to any number, we move to the right along the number line. For example:

$$1 + 3 = 4$$




If we add a negative number to any number, we move to the left along the number line. So, adding (−5) is moving 5 units to the left on the number line — which is the same as subtracting 5. For example:

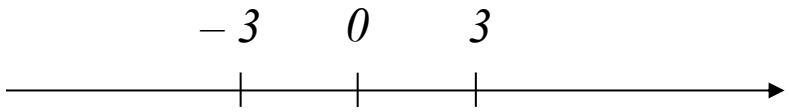
$$1 + (-5) = 1 - 5 = -4$$



**Opposites.**

Pairs of numbers -1 and 1, -2 and 2, -3 and 3 etc. are called the opposites. They lie at the same distance from zero on the number line, but in the opposite directions. For any number A (whether positive or negative), the number denoted by **−A is the opposite of A**. For example, −(−3) is the opposite of (−3), which is equal to 3. So

$$-(-3) = 3$$




What about subtracting a negative number?

For example:

$$1 - (-2) = ?$$

We know that −(−2) is the opposite of negative 2, which is equal to 2. So,

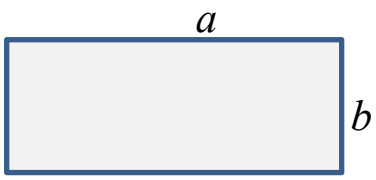
$$1 - (-2) = 1 + 2 = 3$$

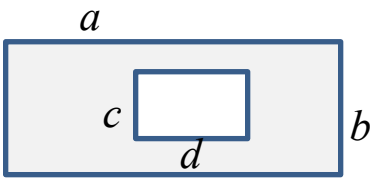
**Homework**.

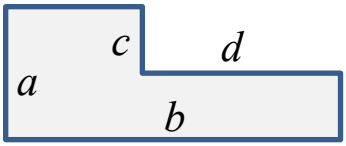
1. Alex is $m$ years old. Robert is $n$ years older than Alex.
   a) How old will the boys be in 3 years?
   b) How many times Robert will be older than Alex in 3 years?
   Solve the problem for $m = 2, n = 10$.

2. Julia had 20 cards. She gave $a$ cards to her sister. How many cards she has now? Can $a$ be any number?

3. Write the expressions for the shaded areas below (all angles are right angles):




4. Fill up the table:

| $a$ | 7 | $-4$ | | | 5 | | 0 | |
|---|---|---|---|---|---|---|---|---|
| $-a$ | | | 0 | $-1$ | | 8 | | $-3$ |

5. Compare:

$-4 \quad 4$ $\qquad$ $6 \quad -4$ $\qquad$ $\frac{2}{3} \quad -\frac{3}{2}$

$-4 \quad -2$ $\qquad$ $-4 \quad 0$ $\qquad$ $-\frac{2}{3} \quad -1$

6. Compute:

$3 + (-2) =$ $\quad$ $3 + (2) =$ $\quad$ $-3 - (-2) =$

$3 - (2) =$ $\quad$ $-3 + (-2) =$ $\quad$ $-3 + (2) =$

$3 - (-2) =$ $\quad$ $-3 - (2) =$ $\quad$ $-3 + (3) =$

7. Fill the empty spaces in the table:

| $\boldsymbol{c}$ | $\boldsymbol{b}$ | $\boldsymbol{b \cdot c}$ |
|---|---|---|
| $\frac{3}{8}$ | $\frac{3}{4}$ | |
| $\frac{3}{4}$ | | $\frac{9}{21}$ |
| | $\frac{2}{3}$ | $\frac{16}{21}$ |

8. Write without parenthesis:

Example:

$-(-3) = 3$ $\quad$ $-(+7) = -7$

$a.\ -(11)$ $\quad$ $b.-(9)$ $\quad$ $c.\ -(-7)$

$d.\ -(-10)$ $\quad$ $e.-(15)$ $\quad$ $f.\ -(-20)$

9. Each floor of a residential building has $f$ two-bedroom apartments and $g$ three-bedroom apartments. The building has 5 floors. How many apartments are there in the building? Write the expression with variables, then solve the problem for $f = 3$ and $g = 4$

10. Create your own problems, which can be solved by the following expressions, give some values to the variables, and solve your problems quantitively:

$a.\ x - y$ $\quad$ $b.\ c + 3c$ $\quad$ $c.\ k{:}9$