Math 4b. Classwork 19

Exponent

Exponentiation is a mathematical operation, written as a^n, involving two numbers, the **base** *a* and the **exponent** *n*. When *n* is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, a^n is the product of multiplying *n* bases:

$$a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$$

In that case, a^n is called the ***n*-th power of *a***, or ***a* raised to the power *n***.

The exponent indicates how many copies of the base are multiplied together. For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the *base*, 5 is the *exponent*, and 243 is the *power* or, more specifically, *the fifth power of 3*, *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

1. If the same base raised to the different power and then multiplied:

$4^3 \cdot 4^5 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^8 = 4^{3+5}$

Or in a more general way:

$$a^n \cdot a^m = \underbrace{a \cdot a \dots \cdot a}_{n\ times} \cdot \underbrace{a \cdot a \dots \cdot a}_{m\ times} = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n+m\ times} = a^{n+m}$$

2. If the base raised to the power of *n* then raised again to the power of *m*:

$$(4^3)^5 = (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3) \cdot (4^3)$$
$$= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)$$

Or in a more general way:

$$(a^n)^m = \underbrace{a^n \cdot a^n \cdot \dots \cdot a^n}_{m\ times} = \underbrace{\underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times} \cdot \dots \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{n\ times}}_{m\ times} = a^{n \cdot m}$$

3. If we want to multiply $5^n = \underbrace{5 \cdot 5 \cdot 5 \ldots \cdot 5}_{n\ times}$ by another 5 we will get the following expression:

$$5^n \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n\ times} \cdot 5 = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n+1\ times} = 5^{n+1} = 5^n \cdot 5^1$$

$$5^1 = 5$$

We can write the same property for any number:

$$a^n \cdot a = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \ldots \cdot a}_{n+1\ times} = a^{n+1} = a^n \cdot a^1$$

In order to have the set of power properties consistent, $a^1 = a$ for any number a.

4. If there are two numbers a and b:

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot \ldots \cdot (a \cdot b)}_{n\ times} = \underbrace{a \cdot \ldots \cdot a}_{n\ times} \cdot \underbrace{b \cdot \ldots \cdot b}_{n\ times} = a^n \cdot b^n \qquad (9)$$

- $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
- $a^n \cdot a^m = a^{n+m}$
- $(a^n)^m = a^{n \cdot m}$
- $(a \cdot b)^n = a^n \cdot b^n$

Homework

1. Continue the sequence:

   $a.\ 1, 4, 9, 16 \ldots$ $b. 1, 8, 27, \ldots$ $c.\ 1, 4, 8, 16 \ldots$

2. Calculate:

   Example: $2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

   $(2 \cdot 3)^3;\quad 2 \cdot 3^3;\quad \frac{1}{3^2};$

   $(-5)^2;\quad -5^2;\quad (-3)^3;$

3. Represent numbers as a power of 10:

   Example: $1000^3 = (10^3)^3 = 10^{3 \cdot 3} = 10^9$

   $100^2;\quad 100^3;\quad 100^4;\quad 100^5;\quad 100^6;$

4. Write the number which extended form is written below;

   Example: $2 \cdot 10^3 + 7 \cdot 10^2 + 2 \cdot 10 + 6 = 2726;$

   $a)\ 2 \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 8;$ $b)\ 7 \cdot 10^3 + 2 \cdot 10^2 + 0 \cdot 10 + 1;$

   $c)\ 9 \cdot 10^3 + 3 \cdot 10 + 3;$ $e)\ 4 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10 + 4;$

5. Write the following expressions in a shorter way replacing product with power:

   Examples:

   $(a) \cdot (a) \cdot (a) \cdot (a) = (a)^4,$ $3m \cdot m \cdot m \cdot 2k \cdot k \cdot k \cdot k = 6m^3k^4$

   1) $(ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab) \cdot (ab);$ 2) $(5m)(5m) \cdot 2n \cdot 2n \cdot 2n;$

   3) $a \cdot b \cdot b \cdot b \cdot b \cdot b;$

6. Write the following numbers as a second power of a number:
   Example: $25 = 5^2$
   100, 121, 144, 225,

7. What should be the exponent for the equation to hold?
   Example: $8^* = 512$
   Answer: $8^3 = 512$
   $a)\ 2^* = 64;$ $b)\ 3^* = 81;$ $c.)\ 7^* = 343$

8. Come up with the problem about the distance between two objects, that can be solved by the formula, and solve it.
   Example: $d = 500 - 2.5(70 + 30)$
   Problem: Two cities are 500 miles apart. A bus and a car started moving toward each other. Speed of the car is 70 m/h, speed of the bus is 30 m/h. What would be the distance between them in 2.5 hours?
   $d = 500 - 2.5(70 + 30) = 500 - 2.5 \cdot 100 = 250\ miles$

   1) $d = 18 + (16 + 4) \cdot 3$
   2) $d = 96 - 4 \cdot (56 - 40)$