

MATH 5: HANDOUT 11
BINARY NUMBERS. n -ARY NUMBERS.

Today we talked more about binary numbers. We discussed arithmetic operations with binary numbers. We also touched on Roman numerals and other bases. For example, in base 3, we only use digits 0, 1, 2, and they correspond to powers of 3:

$$21021_3 = 2 \cdot 81 + 1 \cdot 27 + 0 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 = 196_{10}$$

The topic we didn't discuss (but also an interesting topic) is how letters and other symbols are written as sequences of 0 and 1 (bits) in computers; since there are 2^n such sequences of length n , and there are 26 letters in English alphabet, we need at least 5 bits ($2^5 = 32$) for each English letter. If we want to have lower- and upper-case letters, punctuation, numbers, accented letters such as é, we need more; in real life, people use 8 bits per symbol (called "byte").

The correspondence between actual letters and their codes, i.e. sequences of 0 and 1, is called *encoding*. The most common encoding is called Latin 1 (or ISO 8859-1). It is shown in the table below. In this table, rows correspond to the last 4 bits, and columns, to the first four bits. For example, lower case letter "a" has code 01100001.

				b ₈	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	
				b ₇	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	
				b ₆	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	
				b ₅	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	
b ₄	b ₃	b ₂	b ₁	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15		
0	0	0	0	00			SP	0	à	P	`	p			NBSP	°	À	Ð	à	ø	0
0	0	0	1	01			!	1	A	Q	a	q			í	±	Á	Ñ	á	ñ	1
0	0	1	0	02			"	2	B	R	b	r			¢	²	Â	Ò	â	ò	2
0	0	1	1	03			#	3	C	S	c	s			£	³	Ã	Ó	ã	ó	3
0	1	0	0	04			\$	4	D	T	d	t			¤	'	Ä	Ô	ä	ô	4
0	1	0	1	05			%	5	E	U	e	u			¥	µ	Å	Õ	å	õ	5
0	1	1	0	06			&	6	F	V	f	v			¦	¶	Æ	Ö	æ	ö	6
0	1	1	1	07			'	7	G	W	g	w			§	-	Ç	×	ç	÷	7
1	0	0	0	08			(8	H	X	h	x			"	,	È	Ø	è	ø	8
1	0	0	1	09)	9	I	Y	i	y			©	¹	É	Ù	é	ù	9
1	0	1	0	10			*	:	J	Z	j	z			ª	º	Ê	Ú	ê	ú	A
1	0	1	1	11			+	;	K	[k	{			«	»	Ë	Û	ë	û	B
1	1	0	0	12			,	<	L	\	l				¬	¼	Ì	Ü	ì	ü	C
1	1	0	1	13			-	=	M]	m	}			SHY	½	Í	Ý	í	ý	D
1	1	1	0	14			.	>	N	^	n	~			®	¾	Î	Þ	î	þ	E
1	1	1	1	15			/	?	O	_	o				¯	¿	Ï	ß	ï	ÿ	F
				0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	hex	

HOMEWORK

1. What is the largest number that can be written as a 5-digit binary number? (Hint: what is the smallest 6-digit binary number?)
2. Is it possible to encode every letter of English alphabet by a 4-digit binary number? You can choose any way you like — for example encoding A as 0000, B as 0001, or by any other method.
Would it be possible if we used 5-digit binary numbers?
3. Write numbers 45 and 165 in binary system
4. A car has traveled 125 miles during some period. During the same period, another car, which is faster by 10 mph, has traveled 150 miles. What is the speed of the faster car?
5. Do the following arithmetic operations with binary numbers. Try doing them without converting the numbers to decimal form.

(a) $110101_b + 111011_b$

(b) $10101_b \times 1011_b$

(c) $(10101_b + 1101_b) \times 10110_b$

6. There are five heads and fourteen legs in a family. How many people and dogs are in the family?
7. Fish head weighs as much as the tail and half of the body together. The body weighs as much head and tail together, and the tail weighs 1 kg. How heavy is the fish?
8. The following is a beginning of a computer file. Can you decode it (assuming it is written in the standard, Latin 1, encoding)?

01010100 01101111 01110000 00100000 01110011 01100101 01100011 01110010 01100101 01110100 00001010