# MATH 5: HANDOUT 15
# SQUARE ROOT. PYTHAGOREAN THEOREM. DIFFERENCE OF SQUARES.

## SQUARE ROOTS

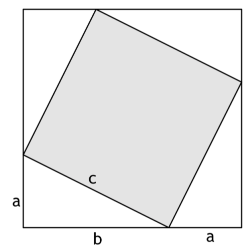
Square root of $a$ is a number whose square is equal to $a$. For example: square root of 25 is 5, because $5^2 = 25$.

- Notation: square root of number $a$ is commonly denoted $\sqrt{a}$.
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$, but $\sqrt{a+b}$ is **not** equal to $\sqrt{a} + \sqrt{b}$.

**Pythagorean theorem:** In a right triangle with legs $a, b$ and hypotenuse $c$, one has

$$a^2 + b^2 = c^2 \qquad \text{ot} \qquad c = \sqrt{a^2 + b^2}.$$

**Proof:** Consider the following picture:




In this square, the total area is

$$(a + b) \times (a + b) = a \times (a + b) + b \times (a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

On the other hand, the area of each triangle is $\frac{1}{2}ab$, and the area of shaded square is $c^2$. Thus, we get

$$a^2 + 2ab + b^2 = 4 \times \tfrac{1}{2}ab + c^2$$

which gives $a^2 + b^2 = c^2$. □

For example, in a square with side 1, the diagonal has length $\sqrt{2}$.

It is possible — but not easy — to find a right triangle where all sides are whole numbers. The easiest such triangle is the triangle with sides $3, 4, 5$.

## HOMEWORK

**1.** Find the following square roots. If you can not find the number exactly, at least say between which two whole numbers the answer is, e.g., between 5 and 6.
   (a) $\sqrt{16}$
   (b) $\sqrt{81}$
   (c) $\sqrt{10,000}$
   (d) $\sqrt{10^8}$
   (e) $\sqrt{50}$

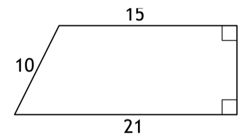
**2.** If, in a right triangle, one leg has length 1 and the hypotenuse has length 2, what is the other leg?

**3.** Can you find a right triangle where all sides are whole numbers and the hypotenuse is 13?

**4.** Find
   (a) $\sqrt{2^6 \times 7^2}$;
   (b) $\sqrt{\frac{1}{16}}$;

(c) $\sqrt{\frac{4}{9}}$;

**5.** Find the height and area of the figure below. Lengths of three sides are given; the two marked angles are right angles.




**6.** The side of an equilateral triangle is $1$ m. Find its height and the area.

**7.** Can you find whole numbers $a, b$ such that $a^2 - b^2 = 17$?

**8.** * Take some positive number $x < 100$ and using calculator (or computer) calculate the number $\frac{x}{2} + \frac{1}{x}$. Call the result $x$ and repeat the same calculation with the new $x$. Do it 10 times. Then take the result and square it. What did you get? Try to do the same thing starting with different numbers. Is it surprising?