

MATH 5: HANDOUT 24 GEOMETRY 5.

Area

Recall that the area of a rectangle is (length) \times width, and area of a right triangle with legs a, b is $\frac{1}{2}ab$ (because putting together two such triangles we get a rectangle with sides a, b). Here are more formulas:

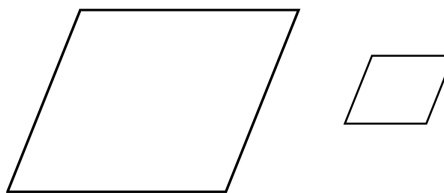
Area of a triangle with base b and height h is $\frac{1}{2}bh$

Area of a parallelogram with base b and height h is bh

Area of a trapezoid with bases a, b and height h is $h \times \frac{a+b}{2}$

Similar figures

Two figures are called *similar* if one of them can be obtained from the other by rescaling, or stretching (equally in all directions).



In similar figures all angles are the same, and all lengths are increased in the same proportion. For example, if triangles ABC and $A'B'C'$ are similar, then

$$\angle A = \angle A', \quad \angle B = \angle B', \quad \angle C = \angle C'$$

$$A'B' = kAB, \quad A'C' = kAC, \quad B'C' = kBC$$

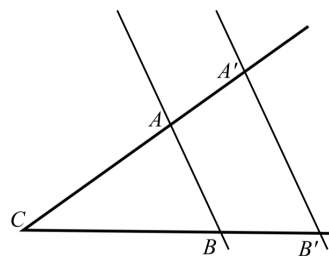
The number k is called the **similarity coefficient**.

SIMILARITY TEST FOR TRIANGLES

AAA Rule. *If three angles of one triangle are equal to corresponding angles of another triangle, then the triangles are similar.*

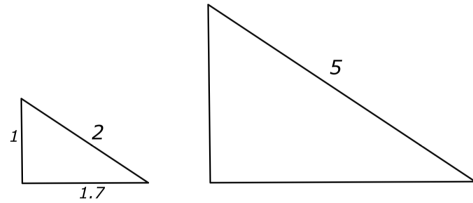
For example, if ABC , $A'B'C'$ is a right triangles with $\angle C = \angle C' = 90^\circ$, $\angle A = \angle A' = 30^\circ$, then these triangles are similar.

Another example is shown in the figure to the right. Let lines $l = AB$, $l' = A'B'$ be parallel. Then triangles ABC and $A'B'C$ are similar: indeed, by the theorem about alternate angles, we have $\angle A = \angle A'$, $\angle B = \angle B'$.

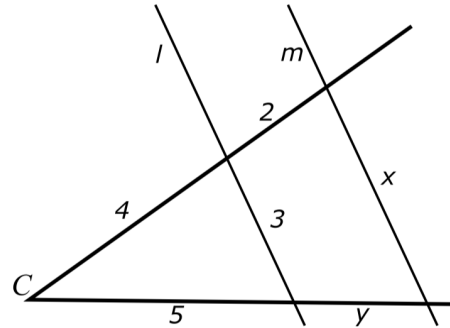


Homework

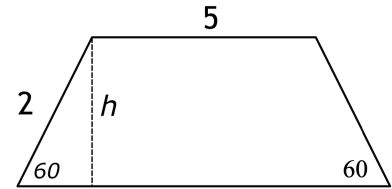
1. Find the missing lengths in the second triangle (triangles are similar).



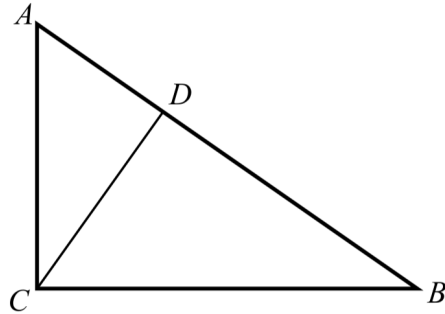
2. Find the lengths x, y in the figure (lines l, m are parallel).



3. The figure to the right shows some of the angles and sides in a trapezoid. The height h of this trapezoid is equal to $\sqrt{3}$. Find all other angles, sides and area of the trapezoid. (Hint: you will need Pythagorean theorem!)



4. The figure shows right triangle ABC : $\angle C = 90^\circ$, with $AC = 4$ cm, $AB = 5$ cm. The line CD is the altitude, i.e., it is perpendicular to side AB .
- Show that triangles ABC and ACD are similar.
 - Find lengths BC, CD .



5. Given a rectangle with sides 2 cm and 4 cm, can you:
- Cut it into three pieces that can be rearranged to get a right triangle?
 - Cut it into three pieces that can be rearranged to get a square?
6. In a rhombus, two diagonals are 16 cm and 12 cm. Find the area and perimeter of the rhombus.
7. Solve the equation $7x - 11 = 5x - 3$.