MATH 5: HANDOUT 24
GEOMETRY 4.

Special quadrilaterals: parallelogram, rhombus, trapezoid

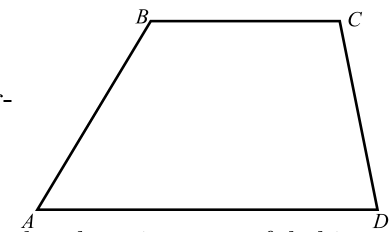
Recall that a **parallelogram** is a quadrilateral in which opposite sides are parallel. Here are some of the properties of parallelograms; all of them were either proved before or will be proved in this homework.

1. In a parallelogram, opposite sides are equal. Conversely, if $ABCD$ is a quadrilateral in which opposite sides are equal: $AB = CD$, $BC = AD$, then $ABCD$ is a parallelogram.

2. In a parallelogram, diagonals bisect each other, i.e. the intersection point of two diagonals is the midpoint of each of them. Conversely, if $ABCD$ is a quadrilateral in which diagonals bisect each other, then $ABCD$ is a parallelogram.

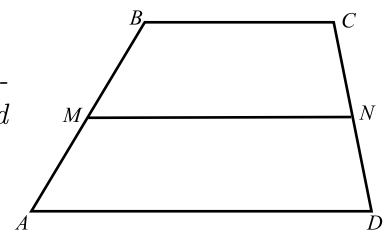
A **rhombus** is a quadrilateral in which all four sides are equal. By property 1 above, any rhombus is also a parallelogram, so all the above properties hold. In addition, in a rhombus diagonals are perpendicular (shown in previous homework).

A **trapezoid** is a quadrilateral in which one pair of opposite sides are parallel: $AD \parallel BC$. These parallel sides are usually called **bases**.

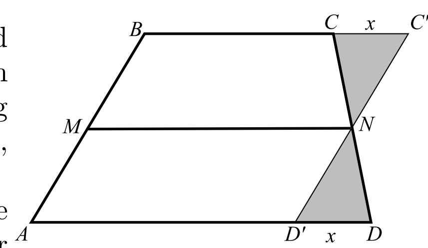


A trapezoid does not have as many useful properties as a parallelogram, but here is one useful thing.

Theorem. *Let $ABCD$ be a trapezoid with bases AD, BC. Let M be midpoint of side AB and N — midpoint of side CD. Then $MN \parallel AD$ and $MN = \frac{AD+BC}{2}$.*



Proof. In homework exercise 2, you will show that it works for a parallelogram. Let us use it.
Draw a line $C'D'$ through point N parallel to AB. Then the two shaded triangles are congruent by ASA, so N is also the midpoint of $C'D'$. On the other hand, $ABC'D'$ is a parallelogram, so MN is the line connecting midpoints of two sides of a parallelogram. Thus, by exercise 2, $MN \parallel AD$, $MN = AD' = BC'$.
Denote $x = CC' = BB'$. Then $BC' = BC + x$, $AD' = AD - X$. Since $ABC'D'$ is a parallelogram, $BC' = AD'$, so $BC + x = AD - x$. Solving for x, we get $x = \frac{AD-BC}{2}$, so $MN = BC' + BC + x = \frac{AD+BC}{2}$. □



Area

Recall that the area of a rectangle is (length)×width, and area of a right triangle with legs a, b is $\frac{1}{2}ab$ (because putting together two such triangles we get a rectangle with sides a, b). Here are more formulas:

Area of a triangle with base b and height h is $\frac{1}{2}bh$

Area of a parallelogram with base b and height h is bh

Area of a trapezoid with bases a, b and height h is $h \times \frac{a+b}{2}$