MATH 5: HOMEWORK 24
GEOMETRY 4.

Warning: in this homework, more than ever, you will need results of previous exercises when doing the next one. So when doing, say, exercise 2, see if you can make use of exercise 1.

1. Let $ABCD$ be a quadrilateral such tath $AB = CD$, $AB \parallel CD$. Show that then $ABCD$ is a parallelogram. [Hint: show that tirangles $\triangle ABD$, $\triangle CDB$ are congruent.]

2. Let $ABCD$ be a parallelogram, and let M, N be midpoints of sides AB, CD. Show that then $AMND$ is a parallelogram, and deduce from this that $MN \parallel AD$, $MN = AD$.

3. (a) Show that if in a quadrilateral $ABCD$ diagonals bisect each other (i.e., intersection point is hte midpoint of each of the diagonals), then $ABCD$ is a parallelogram. [Hint: find some congruent triangles in the figure.]

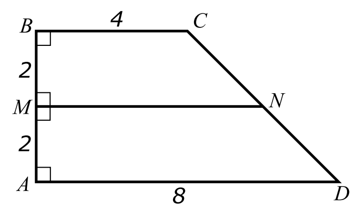
(b) Show that if in a quadrilateral $ABCD$ diagonals bisect each other and are perpendicular, then it is a rhombus.

4. To check whether a piece of paper is a square, John folds it along a diagonal. If the corners match, he decides it is a square. Is he right? What if he folds along both diagonals?

5. Can you cut a trapezoid into pieces from which you can construct a rectangle?

*6 Let ABD be a triangle, and M, N —midpoints of sides AB, BD. Show that then $MN \parallel AD$, $MN = \frac{1}{2}AD$. [Hint: think of the triangle as a trapezoid in which the top base is so small it becomes a single point. Try to see if the proof given above for trapezoids will work for a triangle, too.]

7. Find all lenghts, angles, and area in the figure shown to the right.



8. Suppose you have a large supply of tiles, all of the same size and shape — namely, a parallelogram. Can you tile a plane with these tiles? Can you find different ways of doing this? What if instead of a parallelograms you have trapezoids?