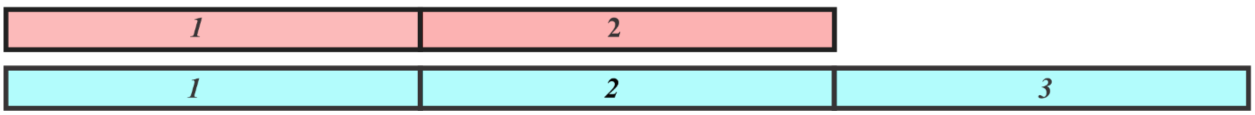
Classwork 6. Algebra.

Algebra.

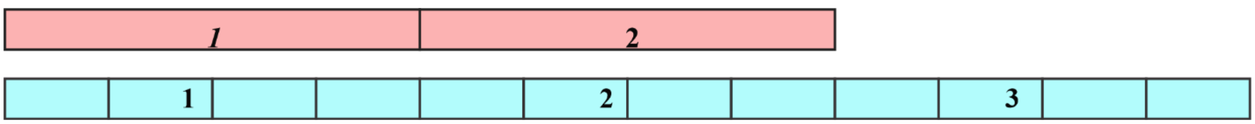
Ratio and proportions.

Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



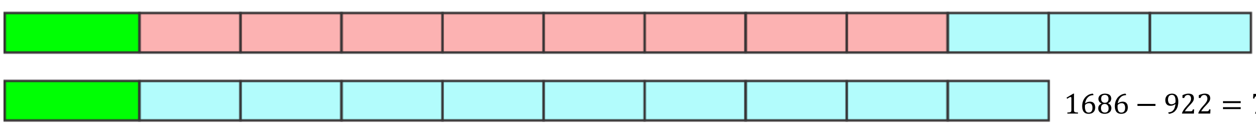
We took as “unit” a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)



Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can’t divide 3 “units” into 4 parts, without getting fractions. So, let’s find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.

Step 3. Let’s compare the number of sold and leftover balloons.

922



$1686 - 922 = 764$

Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small “units” of blue balloons are $922 - 764 = 158$, or one such “unit” is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations?

Let’s try.

$G + B + R = 1686$

$3R = 2B$

$\frac{1}{2}G + R + \frac{1}{4}B = 922$

$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$

$R - \frac{1}{2}B = 158$

$$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$$

Direct and invers proportionality.

Last week we discuss proportions, two equal ratios. What can we say if there are many equal ratios? For example, If I walk with constant speed, how the time of my walk corelated with the distance I walked?

Fill the table:

My speed is 3 km per hour

t	0.5	1	1.5	2	2.5	3	3.5	4
S								

My speed is 5 km per hour

t	0.5	1	1.5	2	2.5	3	3.5	4
S								

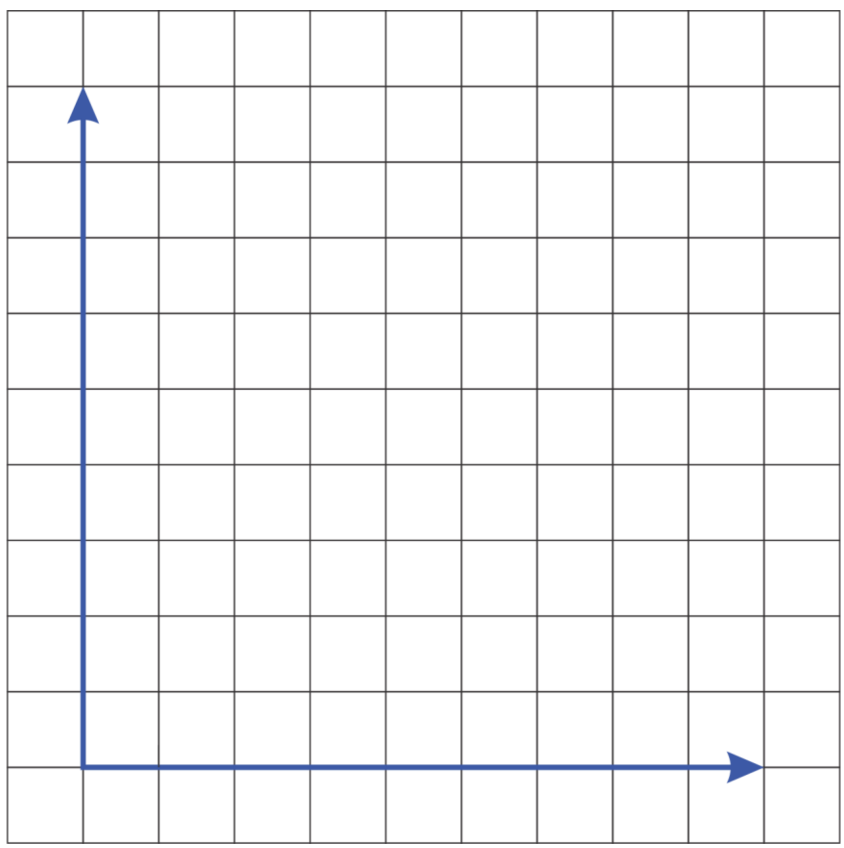
$$\frac{S}{t} = v$$

As you can see, if the speed is constant, the ratio of a distance to time is always be the speed.

In other words, $S = 5\left({}^{km}/_{h}\right) * t(hour)$, 5 is a constant, and it's called a constant of proportionality. Longer travel $\Rightarrow$ further from the initial point, and the ratio between these two variables will be always the same, *distance*:*time* is speed. If the time three time longer, the distance will be three time greater as well. (When the speed is constant of cause). Two variables, *distance* and *time*, are dependent from each other, they are in the relation of proportionality.

We can write the relationship between the distance, time, and speed as

$$S = v \cdot t$$



Two variables (y and x) are related proportionally if

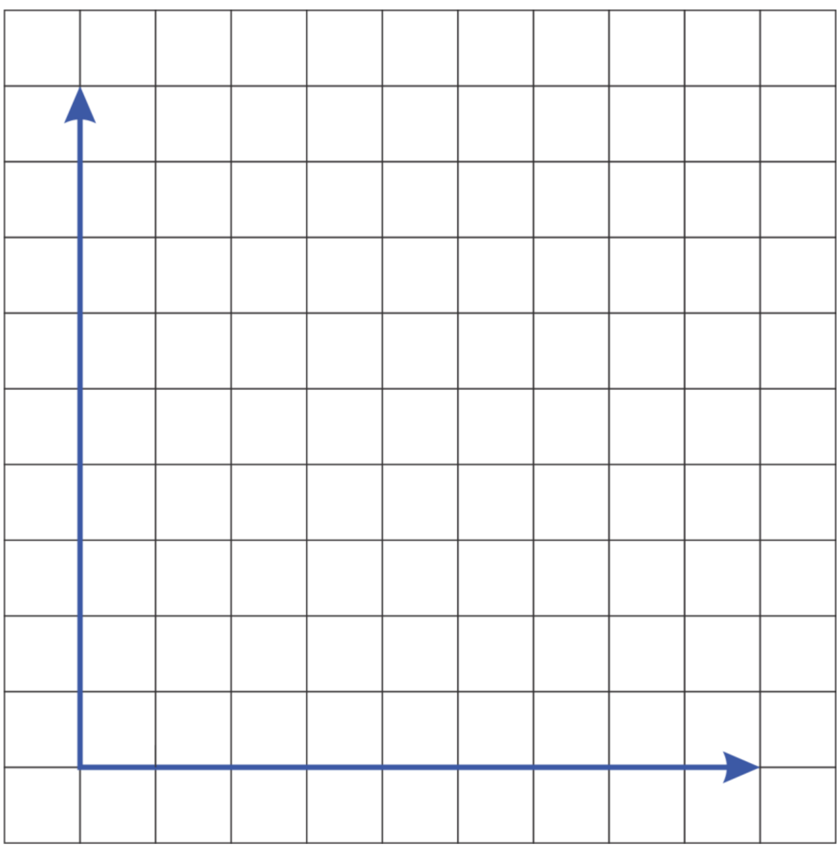
$$y = kx; \quad \frac{y}{x} = k, x \neq 0$$

We say that y is directly proportional to x, and coefficient of proportionality is k. Another example of direct proportionality is the circumference and the diameter of a circle, π is coefficient.

A notebook cost 3 dollars. How much I need to pay if I buy 3 notebooks, 5, 12? What variables are used, what is the relationship between them. Can a constant of proportionality be found?

The distance between my home and my work is 20 miles. Let's see, how the time and speed are corelated, if I ride a bike to work:

v	80	40	20	10	5	2	1	0.5	0.1
t									



This kind of correlation is

Exercises:

1. A squirrel is doing a stock of acorns for winter. Every 20 minutes it brings 2 acorns. How many acorns it will have in 40 minutes? 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of acorns				

2. Bacteria are dividing every 20 minutes. I want to make yogurt and I put 1 bacterium in a cup of milk. How many bacteria will be there in the milk in 20 minutes? in 40 minutes? in 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of bacteria				

In which problem the variables are proportional?

3. The relationship between two variables is given in the table below. Is this relationship proportional? If so, what is the constant of proportionality?

a.

x	9	15	33	45	66
y	3	5	11	15	22

b.

x	3	2	5	4	6
y	9	4	25	16	36

c.

x	3	2	1	$\frac{1}{3}$	30
y	1	$\frac{3}{2}$	3	9	0.1

4. Are the following variables proportional?
   a. Speed and time of movement on a distance of 50 km.
   b. Speed and corresponding distance after 2 hours of driving.
   c. Price of the 1 notebook and the number of notebooks which can be bought with 24 dollars.

d. Length and the width of the rectangle with the area of 60 cm^2.

5. A car travels 60 km during a certain time. How this time will change, if the speed will be increased 3 times?

6. Which of the following formulas describe the direct proportionality, inverse proportionality or neither of the two?

$$P = 5.2b; \quad K = \frac{n}{2}; \quad a = \frac{8}{b}; \quad M = m:5; \quad G = \frac{1}{4k};$$
$$a = 8q + 1; \quad c = 4:d, \quad 300 = v \cdot t; \quad ab = 18; \quad S = a^2$$

7. Peter's time of the driving to work usually is 1 hour and 20 minutes. Yesterday was a bad weather and Peter reduced his speed by 10 km/h and reached his work in 1.5 hours. What is the distance between Peter's house and his work?

8. Anna drove her car 345 miles and used 15 gallons of gasoline. At the same rate, how many miles could she drive her car using 35 gallons of gasoline? Set up and solve a proportion.

9. *y* is directly proportional to *x*. Given that y = 144 and x = 12. Find the value of y when x = 7.