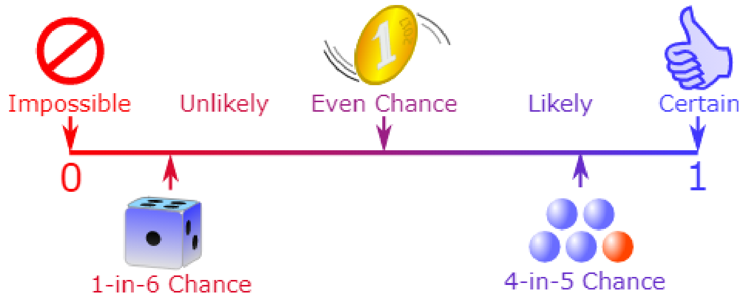
Classwork 25.

Probability.



What will you get if you toss a coin? Obviously, there will be either head or tail. If we would toss this coin many times, how many heads and how many tails we will register? The ratio of the desired outcome to the total number of possible outcomes is the probability of desired outcome to happen. In the example of a tossed coin there are two possible outcomes, head and tail, so the probability to get a head is (if it’s a fair coin)

$$1 \ to \ 2, \quad or \ \frac{1}{2}; \quad or \ 0.5, \quad or \quad 50\%.$$

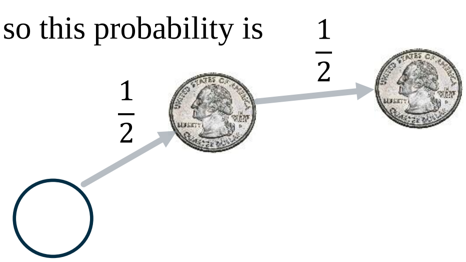
It doesn’t mean that if you flip the coin twice you will definitely get a head.
Rather if you toss the coin 1000 times the head will appear about 500 times. More tossing, the closer the ratio is to ½. Let’s check it!

If we roll a die (dice can be used as singular or plural, die is used only as singular), there are 6 possible outcomes, 1, 2, 3, 4, 5, and 6.
The probability to get 1 is $\frac{1}{6}$ (there is only 1 way to get desirable outcome and 6 possible outcomes.

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

What is a probability to get an even number on a die?
There are 3 possible ways to get even: 2, 4, 6. And 6 total outcomes, so this probability is $3:6 = 0.5$

Let’s toss a coin twice. What is a probability to get head both times?



We can look at this event (get head twice) in two different ways:
First:
Probability to get a head first is ½. The probability to get second head is also ½. The probability to get two heads in a row is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Second:
There are 4 possible outcomes for two tosses:
HH, HT, TH, TT and only one (HH) possibility for us to get HH.

What is a chance to have a jackpot ticket in a lottery of 6/49? In another words, what is a probability that 6 numbers, chosen by the lottery will appear on the thicket?
The probability of the event is a ratio of numbers of the ways this event can happen to the total number of possible outcomes. In the case of lottery, there is only one winning set of six numbers. How many possible outcomes are there? We already know how to calculate it:

First number we can chose from 49, second number from 48 and so on.
To calculate the number of permutations $P(49,6) = 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44$
General calculation of permutation

$$P(n,m) = n(n-1)(n-2) \cdot \ldots \cdot (n-m+1)$$

In our lottery example last factor of the expression for permutation is $44 = 49 - 6 + 1$.

$$P(n,m) = n(n-1)(n-2) \cdot \ldots \cdot (n-m+1) =$$

$$\frac{n(n-1)(n-2) \cdot \ldots \cdot (n-m+1) \cdot (n-m)(n-m-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1}{(n-m)(n-m-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1} = \frac{n!}{(n-m)!}$$

$$P(49,6) = \frac{49!}{(49-6)!}$$

Order of the numbers in the set is not important, so we have to divide the number of permutations (order matter) by the number of permutations inside the group of 6. There are exactly 6! possible way to rearrange 6 objects

$$P(6.6) = 6 \cdot 5 \cdot \ldots \cdot 2 \cdot 1 = \frac{6!}{(6-6)!}$$

Division by 0! Mathematicians defined 0! as 1 .

$$P(6.6) = 6 \cdot 5 \cdot \ldots \cdot 2 \cdot 1 = \frac{6!}{(6-6)!} = 6!$$

Number of possible combinations of 6 numbers from49 is

$$C(49,6) = \binom{49}{6} = \frac{P(49,6)}{6!} = \frac{49!}{(49-6)!\,6!} = \frac{49 \cdot 48 \cdot \ldots \cdot 3 \cdot 2 \cdot 1}{43 \cdot 42 \cdot 41 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot \ldots \cdot 2 \cdot 1}$$

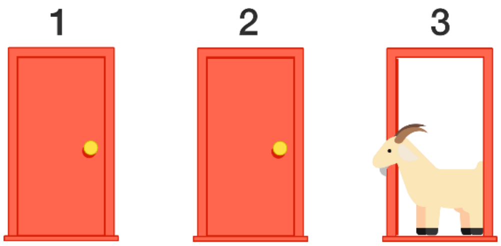
$$= \frac{49 \cdot 48 \cdot \ldots \cdot 44}{6 \cdot 5 \cdot \ldots \cdot 2 \cdot 1} = \frac{49 \cdot 8 \cdot 47 \cdot 23 \cdot 3 \cdot 11}{1} = 13{,}983{,}816$$

Number of combinations

$$C(n,m) = \binom{n}{m} = \frac{P(n,m)}{m!} = \frac{n!}{(n-m)!\,m!}$$

Probability to win a jackpot is $\frac{1}{13{,}983{,}816} \approx 7.15 \cdot 10^{-8}$

Monty Hall problem

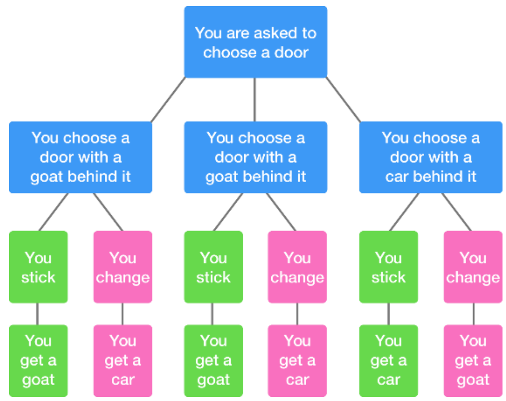


In the problem, you are on a game show, being asked to choose between three doors. Behind each door, there is either a car or a goat. You choose a door. The host, Monty Hall, picks one of the other doors, which he knows has a goat behind it, and opens it, showing you the goat. (You know, by the rules of the game, that Monty will always reveal a goat.) Monty then asks whether you would like to switch your choice of door to the other remaining door. Assuming you prefer having a car more than having a goat, do you choose to switch or not to switch?

1. The host must always open a door that was not picked by the contestant.
2. The host must always open a door to reveal a goat and never the car.
3. The host must always offer the chance to switch between the originally chosen door and the remaining closed door.

There are three possible arrangements of one car and two goats behind three doors and the result of staying or switching after initially picking door 1 in each case

Behind door 1	Behind door 2	Behind door 3	Result if staying at door #1	Result if switching to the door offered
Goat	Goat	**Car**	Wins goat	**Wins car**
Goat	**Car**	Goat	Wins goat	**Wins car**
Car	Goat	Goat	**Wins car**	Wins goat


You are asked to choose a door
You choose a door with a goat behind it
You choose a door with a goat behind it
You choose a door with a car behind it
You stick
You change
You stick
You change
You stick
You change
You get a goat
You get a car
You get a goat
You get a car
You get a car
You get a goat