MATH 6: HANDOUT III
LOGIC I. KNIGHTS AND KNAVES

This week we travel to the famous (among logic puzzle fans) island of Knights and Knaves [1]. On this island, there are only two kinds of people: knights, who always tell the truth, and knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave...

Our job is to use logic in order to determine who is a knight or who is a knave. As we start walking through the island, we run into Mary, an inhabitant of the island. She tells us the following statement:

$$S(\text{Mary}) = \text{I am a knight}$$

Just based on this, can we determine whether Mary is a Knight or a Knave?

M	S(M)
T	
F	

We keep going on our journey and we find Zoe, another inhabitant. After our interaction with Alice, we wonder if it would be possible for Zoe to say the following statement.

$$S(\text{Zoe}) = \text{I am a knave}$$

Z	S(Z)
T	
F	

Later, we run into Alice and Bob. Suddenly, Alice says: "At least one of us is a knave". Can you determine what type of inhabitants are Alice and Bob just from this?

A	B	S(A)
T	T	
T	F	
F	T	
F	F	

[1] *Copyright notice:* most of these problems come from books of Raymond Smullyan. If you liked them, get his books in the library and you will find there many more puzzles of the same sort. You can also find a number of such puzzles online at `http://philosophy.hku.hk/think/logic/puzzles.php`

HOMEWORK

Can you determine who is a knight and who is a knave in all of the following cases?

1. You meet two inhabitants: Sally and Zippy. Sally claims, 'I and Zippy are not the same.' Zippy says, 'Of I and Sally, exactly one is a knight.'
2. You meet two inhabitants: Mel and Ted. Mel tells you, 'Either Ted is a knight or I am a knight.' Ted tells you that Mel is a knave.
3. You meet two inhabitants: Ted and Zippy. Ted says, 'Of I and Zippy, exactly one is a knight.' Zippy says that Ted is a knave.
4. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves."
5. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights."
6. On the island of knights and knaves, you meet two inhabitants: Bart and Ted. Bart claims, "I and Ted are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave."
7. You meet two inhabitants: Betty and Peggy. Betty tells you that Peggy is a knave. Peggy tells you, 'Betty and I are both knights.'
8. You meet two inhabitants: Zed and Peggy. Zed says that Peggy is a knave. Peggy tells you, 'Either Zed is a knight or I am a knight.'
9. You meet two inhabitants: Zed and Alice. Zed tells you, 'Alice could say that I am a knight.' Alice claims, 'It's not the case that Zed is a knave.'
10. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B, C, D, E).
    A says: "exactly one of us is a Knight"
    B says: "exactly two of us are Knights"
    C says: "exactly three of us are Knights"
    D says: "exactly four of us are Knights"
    E says: "all five of us are Knights"
    Can you find out which of them are Knights?
11. You are in a maze on the island of knights and knaves. There are two doors: you know that one leads to freedom and one leads to certain doom. There are two guards nearby, and you happen to know that one is a knight and one is a knave, but you don't know who is who. They allow you to ask one of them a single question before you choose a door — what do you ask?