MATH 6: HANDOUT VI
LOGIC IV. COMPUTER LOGIC

BASIC LOGIC OPERATIONS

For you convenience, here is the list of logic operations we have used so far:

- NOT A: true if A is false, false if A is true
- A AND B: true if both A and B are true, false otherwise
- A OR B: true if at least one of A and B is true, false otherwise
- A XOR B: true if exactly one of A and B is true, false otherwise
- A NAND B = NOT(A AND B): false if both A and B are true, true otherwise

IF

Recall that we are studying logic rules, in particular logic rules involving operation $\Rightarrow$ (reads "implies", or "if A then B"). Here are some of the more important rules:

- Statement $A \Rightarrow B$ is false when A is true and B is false; in all other situtations $A \Rightarrow B$ is true.
- $A \Rightarrow B$ and $B \Rightarrow A$ are not equivalent: it is possible that one statement is true and the other is false.
- Contrapositive rule: $A \Rightarrow B$ is equivalent to (NOT B) $\Rightarrow$ (NOT A).

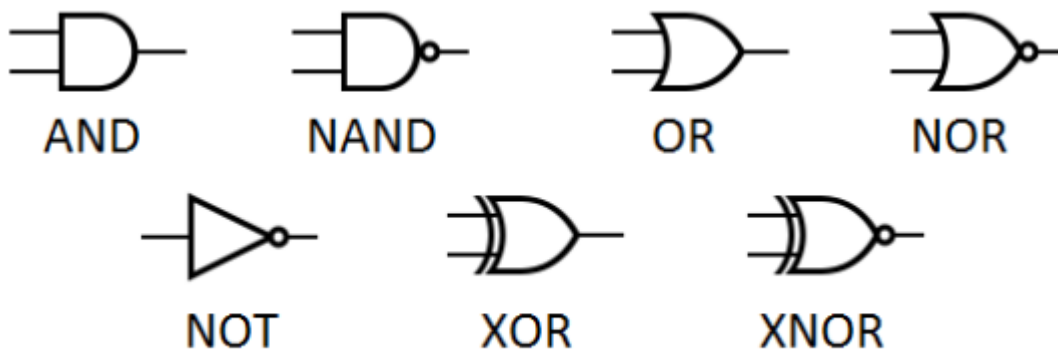This construction is very useful in deducing new results from known ones. Here are some of the rules:

- Given $A \Rightarrow B$ and $B \Rightarrow C$, we can conclude $A \Rightarrow C$
- Given $A \Rightarrow B$ and NOT B, we can conclude NOT A

LOGIC AND COMPUTER CHIPS

We have seen in class that there are computer chips that can reproduce logical operations: each of the inputs and outputs can have voltage 0 or a positive voltage. The usual convention is

- Positive voltage=true
- Zero voltage=false

Then one can relatively easily construct AND, NOT,... chips, and combining them, more complicated chips. The symbols used for the logic gates in electronic circuits are:

1. Simplify the following expressions (do not calculate the exact number, just simplify the exponents as much as possible):

   (a) $\dfrac{6^5 \times 2^4}{3^5 \times 2^2} =$ (b) $(5^2)^7 =$ (c) $(7^2 \times 7^3)^2 =$ (d) $2^{-2} =$
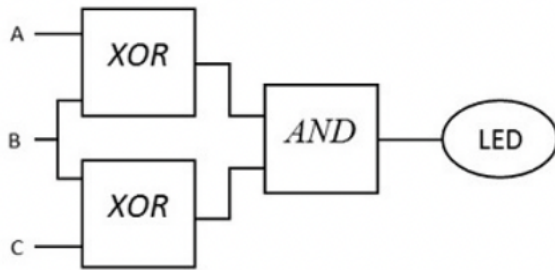
2. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,...

3. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, ...

4. [This problem is due to Lewis Carroll. Some of you may know him as the author of *Alice in Wonderland* and *Through the Looking Glass*. However, he was also a mathematician, and invented a huge number of logical puzzles. This is one of them]

   Babies are illogical. Nobody who can manage a crocodile is despised. Illogical persons are despised. Therefore ...
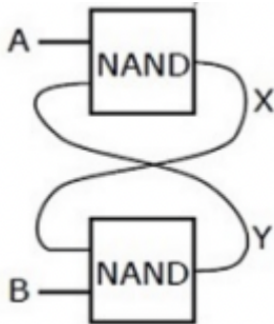
   [Hint: rewrite all of this as statements about some particular person, say X. For example, the first statement would say: if X is a baby, then X is illogical.]

5. The diagram below shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? [Hint: this is the same a writing a truth table for some formula....]
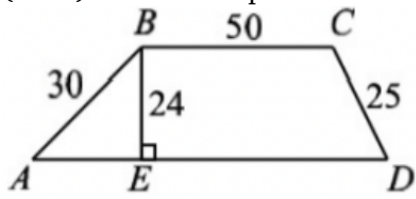

   Note: the wires connecting each of the chips and LED to the power source are not shown.

6. (a) On the island of knights and knaves, you meet two inhabitants, X and Y. X says, "Y is a knave". Y says, "X is a knave". Who is a knave and who is a knight?

   *(b) Consider the circuit below: there are two NAND chips, and output of each one is connected to the input of the other. If inputs A,B are both true (i.e., carry positive voltage), what will be the values of X, Y?

7. (AMC) What is the perimeter of trapezoid ABCD?

8. (AMC) What is the shaded area in the given rectangle?