MATH 6
HANDOUT 4: MORE ABOUT IF

BASIC LOGIC OPERATIONS

For you convenience, here is the list of logic operations we had used:

- NOT A: true if A is false, false if A is true
- A AND B: true if both A and B are true, false otherwise
- A OR B: true if at least one of A and B is true, false otherwise
- A XOR B: true if exactly one of A and B is true, false otherwise

IF

Recall that we are studying logic rules, in particular logic rules involving operation $\Rightarrow$ (reads "implies", or "if A then B"). Here are some of the more important rules:

- $A \Rightarrow B$ and $B \Rightarrow A$ are not equivalent: it is possible that one statement is true and the other is false.
- Contrapositive rule: $A \Rightarrow B$ is equivalent to $(\text{NOT } B) \Rightarrow (\text{NOT } A)$.

This construction is very useful in deducing new results from known ones. Here are some of the rules:

- Given $A \Rightarrow B$ and $B \Rightarrow C$, we can conclude $A \Rightarrow C$
- Given $A \Rightarrow B$ and NOT B, we can conclude NOT A
- Given $A \Rightarrow$ False, we can conclude NOT A (proof by contradiction)

HOMEWORK

In problems 1–3, you need to a)write the obvious conclusion from given statements; and b)justify the conclusion, by writing a chain of arguments which leads to it. It may help to write the given statements and conclusion by logical formulas (denoting the statements which are used by letters $A, B, \dots$ connected by logical operations OR, AND, $\Rightarrow, \dots$).

1. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,...

2. If it is Tuesday and Bill is in a good mood, he goes to his favorite pub, and when he goes to his favorite pub, he comes home very late. Today Bill came home early. Therefore, ...

3. Here is another one of Lewis Carrol's puzzles.
All hummingbirds are richly colored..
No large birds live on honey.
Birds that do not live on honey are dull in color.
Therefore,...

4. Let us consider a new logical operation, called NAND, which is defined by the following truth table:

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

(a) Show that A NAND B is equivalent to NOT(A AND B) (this explains the name: NAND is short for "not and").
(b) Show that A NAND A is equivalent to NOT A.
(c) Write the truth table for (A NAND B) NAND(A NAND B).
(d) Write the truth table for (A NAND A) NAND(B NAND B).
(e) Show that any logical formula which can be written using AND, OR, NOT can also be written using only NAND.

5. On the island of knights and knaves, you meet two inhabitants, X and Y. X says, "Y is a knave". Y says, "X is a knave". Who is a knave and who is a knight?