GEOMETRIC SEQUENCES

A sequence of numbers is a **geometric sequence** or **geometric progression** if if the next number in the sequence is the current number times a fixed constant called the **common ratio** or q.

Example: The sequence $6, 12, 24, 48, \ldots$ is a geometric sequence because the next number is obtained from the previous by multiplication by $q = 2$.

We can also find the n-th term if we know the 1st term and q.

Example: What is a_{10} in the example above?

$$a_1 = 6$$
$$a_2 = a_1 q = 6 \cdot 2 = 12$$
$$a_3 = a_2 q = (a_1 q)q = a_1 q^2 = 6 \cdot 2^2 = 24$$

The pattern is:

$$a_n = a_1 q^{n-1}$$
$$a_{10} = a_1 q^9 = 6 \cdot 2^9 = 6 \cdot 512 = 3072$$

Properties of a Geometric Sequence. Any term is the **geometric mean** of its neighbors:

$$a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$$

Proof:

$$a_n = a_{n-1} q$$
$$a_n = a_{n+1}/q$$

Multipluying these two equalities gives us:

$$a_n^2 = a_{n-1} \cdot a_{n+1}$$

from where we can get what we need.

PROBLEMS

1. Write out the first four terms of each of the following geometric sequences, given the first term b_1 and common ration q.
   (a) $b_1 = 1$ and $q = 3$
   (b) $b_1 = 1$ and $q = \frac{1}{2}$
   (c) $b_1 = -10$ and $q = \frac{1}{2}$
   (d) $b_1 = 27$ and $q = -\frac{1}{3}$

2. What are the first two terms of the geometric progressions $a_1, a_2, 24, 36, 54, \ldots$?

3. Find the common ratio of the geometric progressions $1/2, -1/2, 1/2, \ldots$. What is a_{10}?

4. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50th term?