

MATH 6: HANDOUT 18 COORDINATES I

COORDINATE GEOMETRY: INTRODUCTION

In this section of the course we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the ***x*-axis** and the ***y*-axis**. *x*-axis is usually drawn horizontally, and *y*-axis — vertically. These two axes have a **scale** – “distance” from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point *P*, draw lines through *P* perpendicular to the *x*- and *y*-axes. These lines intersect the axes in points with coordinates x_0 and y_0 . Then the point *P* has *x*-coordinate x_0 , and *y*-coordinate y_0 , and the notation for that is: $P(x_0, y_0)$.

The **midpoint** *M* of a segment *AB* with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

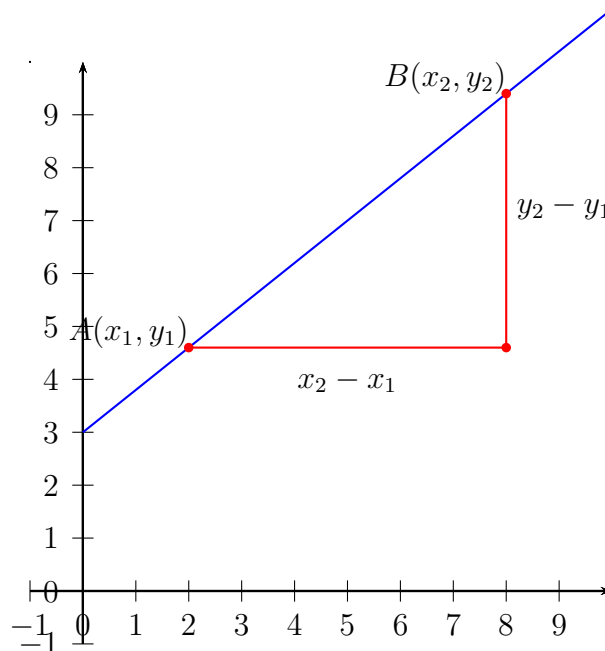
LINES

Given some relation which involves variables *x*, *y* (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form:

$$y = mx + b$$

where *m*, *b* are some numbers, defines a straight line. The slope of this line is determined by *m*: as you move along the line, *y* changes *m* times as fast as *x*, so if you increase *x* by 1, then *y* will increase by *m*:



In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of *y*: $y_2 - y_1$ by the change of *x*: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, b is a ***y*-intercept**, and determines where the line intersects the vertical axis (*y*-axis).

The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

HOMEWORK

1. A point B is 5 units above and 2 units to the left of point $A(7, 5)$. What are the coordinates of point B ?
2. Find the coordinates of the midpoint of the segment AB , where $A = (3, 11)$, $B = (7, 5)$.
3. Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?
4. (a) 3 points $A(0, 0)$, $B(1, 3)$, $D(5, -2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C ?
 (b) 3 points $A(0, 0)$, $B(2, 3)$, $D(4, 1)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C ?
 (c) 3 points $A(0, 0)$, $B(1, 5)$, $D(3, -2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C ?
 (d) Can you guess the general rule: if $A(0, 0)$, $B(b_1, b_2)$, $D(d_1, d_2)$ are 3 vertices of a parallelogram, what are coordinates of point C ?
5. Consider the triangle $\triangle ABC$ with the vertices $A(-2, -1)$, $B(2, 0)$, $C(2, 1)$. Find the coordinates of the midpoint of B and C . Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle $\triangle ABC$.
6. What is the slope of a line whose equation is $y = 2x$? What is the slope of a line whose equation is $y = mx$?
7. In this problem you will find equations that describe some lines.
 - (a) What is the equation whose graph is the y -axis?
 - (b) What is the equation of a line whose points all lie 5 units above the x -axis?
 - (c) Is the graph of $y = x$ a line? Draw it.
 - (d) Find the equation of a line that contains the points $(1, -1)$, $(2, -2)$, and $(3, -3)$.
8. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point $(0, 0)$) and then write the equation of the perpendicular line

(a) $y = 2x$	(b) $y = 3x$
(c) $y = -x$	(d) $y = -\frac{1}{2}x$

Can you determine the general rule: if the slope of a line is k , what is the slope of the perpendicular line?