

MATH 7 HOMEWORK 3: Algebraic operations continued
October 2, 2022

1. Exponents Laws

If a is a real number, n is a positive integer

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

2. Radicals

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. Main Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

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1. Simplify:

$$a) \frac{1}{(x+1)} - \frac{1}{(x-1)}$$

$$b) \left(1 + \frac{1}{x}\right) \div (x+1)$$

$$c) \left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$

2. Simplify, use powers for radicals

$$a. \sqrt{\frac{56}{13}} \cdot \sqrt{\frac{26}{7}} =$$

$$b. \sqrt[3]{\frac{56}{13}} \sqrt[3]{\frac{26}{7}} =$$

$$c. \sqrt{48} =$$

$$d. \sqrt[4]{48}$$

$$e. \frac{\sqrt{48}}{\sqrt{15}} =$$

$$f. \sqrt[3]{a^2 b^3 c^4 d^5}$$

$$g. \sqrt[4]{a^2 b^3 c^4 d^5}$$

$$h. \sqrt[5]{a^2 b^3 c^4 d^5}$$

3. Express the following expressions in the form $2^r 3^s a^m b^n$:

$$a. 8a^3 b^2 (27a^3)(2^5 ab) =$$

$$b. 3^2 (2ab)^3 (16a^2 b^5)(24b^2 a) =$$

$$c. 16a^2 b^3 (6ab^4)(ab^2)^3 =$$

4. Expand as sums of powers of x :

$$a. (2x+5)^2 =$$

$$b. (2-4x)^2 =$$

$$c. (1-2x)^2 =$$

5. Factor (i.e., write as a product) the following expressions:

$$a. 4x^2 + 8xy + 4y^2$$

$$e. \cancel{(x-2)^2 - 10(x-2) + 25}$$

$$b. 9x^2 - 25$$

$$f. 3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$$

$$c. (x-2)^2 - (y+3)^2$$

$$g. \cancel{a^2 - b^2 - 10b - 25}$$

$$d. 256 - a^8 b^8$$

$$h. x^4 + 4 \quad \text{Hint: add and subtract } 4x^2$$

6. Solve the following equations. :

$$a. 5(x+1) = 3x+2$$

$$d. (x-3)(x+4) = 0$$

$$b. (x^2-1)(x+2) = 0$$

$$e. x^2 + 4x = 0$$

$$c. \frac{x+2}{x+3} = 2$$

$$f. x^3 + 4x = 0$$

7. Prove:

$$a. (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$b. (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

c. Find $(a+b)^4, (a-b)^4$ using the previous results