

MATH 7 HOMEWORK 7: Introduction to Combinatorics – Pascal’s triangle
November 6, 2022

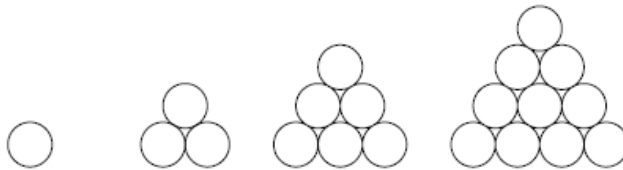
Instructions: Please always write solutions on a *separate sheet of paper*. Solutions should include explanations **how you arrived at this answer**.

1. A dinner in a restaurant consists of 3 courses: appetizer, main course, and dessert. There are 5 possible appetizers, 6 main courses and 3 desserts. How many possible dinners are there?
2. How many ways are there to seat 5 students in a class that has 5 desks? if there are 10 desks?
3. How many ways are there to select first, second and third prize winner if there are 14 athletes in a competition?
4. How many ways are there to put 8 rooks on a chessboard so that no one attacks the others?
5. A dressmaker has two display windows. The left display is for evening dresses and the one in the right window for regular day dresses. Assuming she can put 10 evening dresses in any order, and separately, 5 regular dresses in any order, how many total possibilities of arranging the two display windows are there?
6. The guidelines at a certain college specify that for the introductory English class, the professor may choose one of 3 specified novels, and choose two from a list of 5 specified plays. Thus, the reading list for this introductory class must have one novel and two plays. How many different reading lists could a professor create within these parameters?
7. Finish the chessboard problem (for 8×8-board: how many ways are there to go from lower left corner to upper right corner?)
8. Which of the numbers in Pascal triangle are even? Can you *guess the pattern*, and then carefully explain why it works? (no formulas)
9. What is the sum of all entries in the n-th row of Pascal triangle? Try computing the first several answers and then guess the general formula.
10. What is the alternating sum of all the numbers in n-th row of Pascal triangle, i.e.

$$1 + \binom{n}{1} - \binom{n}{2} + \binom{n}{3} - \dots$$

Try computing the first several sums and then guess the general formula.

11. Let us draw a figure consisting of n rows of circles as shown in the figure below (for n = 1, 2, 3, 4):



Let T_n be the number of circles in n-th figure (for example, $T_1 = 1, T_2 = 3, T_3 = 6 \dots$). These numbers are sometimes called the triangular numbers.

- a. What is the difference $T_{n+1} - T_n$? Try for a few n as 1, 2, 3, 4, 5, ...
- b. Show that the numbers T_n appear in the Pascal triangle as shown below, that is $T_n = \binom{n+1}{2}$. Again, try n = 1, 2, 3, 4 ...

			1		
		1		1	
	1	2		1	
	1	3	3	1	
	1	4	6	4	1
			...		