MATH 7: HOMEWORK 21

Parabolas, adding graphs.

April 2, 2023

1. Quadratic function (revisited +)

Quadratic equation in a standard form: $ax^2 + bx + c = 0$

- a, b, c – coefficients, determinant D: $\boldsymbol{D = b^2 - 4ac,}$ solutions(roots): $\boldsymbol{x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}}$
- **D** determines the number of roots! ($D < 0$ no solutions, $D = 0$ one solution, $D > 0$ two solutions)

Quadratic function in a factored form: $y = a(x - x_1)(x - x_2)$, where

- roots: the numbers x_1 and x_2 – solutions of the quadratic equation ($y = 0$)
- **Vieta's formulas:** The roots are related to the coefficients: $\boldsymbol{x_1 x_2 = \frac{c}{a}}$ and $\boldsymbol{x_1 + x_2 = -\frac{b}{a}}$

Quadratic function in a vertex form: $\boldsymbol{y = a(x - h)^2 + k}$

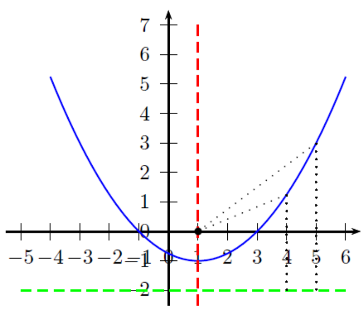
- **Method 1: completing the square**. Use the formulas for fast multiplication.
- **Method 2: find the vertex**. Determine the coefficients a, b, c. Find the vertex x-and y- coordinates

  $\boldsymbol{x_v = h = -\frac{b}{2a}}$. $\boldsymbol{y_v = k} = y(\boldsymbol{x_v}) = a\boldsymbol{x_v}^2 + b\boldsymbol{x_v} + c$

Modified vertex form: rewrite the equation into separate $y -$ and $x -$ part $4\boldsymbol{p}(y - \boldsymbol{k}) = (x - \boldsymbol{h})^2$

Distance from any point on the parabola to focus and directrix: $\boldsymbol{p} = \frac{1}{4a}$

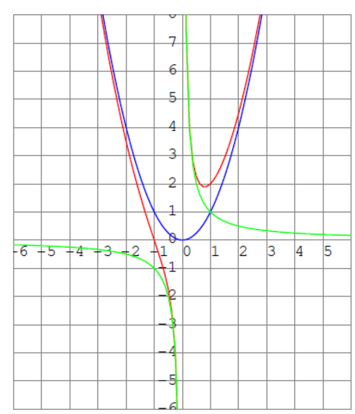
Vertex $V(h, k)$ Focus $F(h, k + \boldsymbol{p})$ directrix $y = k - \boldsymbol{p}$



NEW: Parabola is the set of all points in a plane that are equally distant away from a given point and a given line (see black dotted lines). This given point is called **the focus** (black dot) of the parabola and the line is called **the directrix** (green line).

- If the parabola is of the form $(x - h)^2 = 4p(y - k)$, the vertex is $(h. k)$, the focus is $(h, k + p)$ and directrix is $y = k - p$.

2. Adding Graphs



Now that we know how to draw a lot of basic graphs and how to use transformations, we can draw more complicated graphs — that is, graphs that we get by adding two functions. For example, if we want to draw a graph of a function $y = x^2 + \frac{1}{x}$

We can carefully examine two separate graphs of $y = x^2$ (**blue**) and $y = \frac{1}{x}$ (**green**), and then see what happens if one adds these two graphs (**red**) by adding their y-values for every x.

Homework problems

1. Graph $x^2 = 4y$. What is the focus, directrix and vertex of the parabola?

2. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values. (Review your class notes)
   a. $y = |x| + |x+1|$
   b. $y = |x-1| + |x+1|$
   c. $y = |x-1| - |x+1|$
   d. $|y| = x$

3. Sketch the following functions by first drawing the graph of each addend function and then adding the y-values for a few x-values.
   a. $y = x + \frac{1}{|x|}$
   b. $y = \sqrt{x} + \frac{1}{x}$
   c. $y = x - \frac{1}{x^2}$

4. Find all intersection points of the parabola $y = x^2$ and the circle with radius $\sqrt{6}$ and center at (0,4).