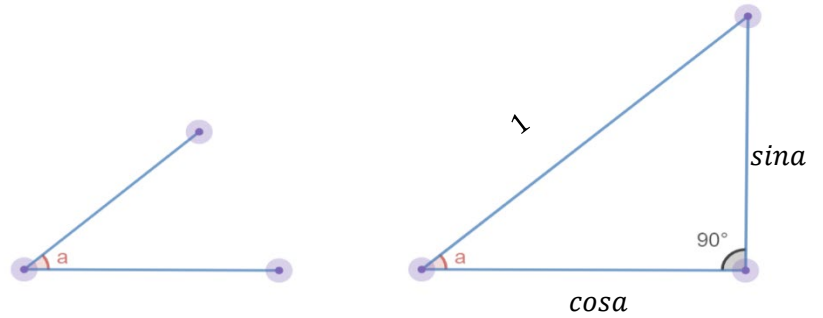


**MATH 7: HOMEWORK 23**  
**Trigonometry, basic definitions.**  
 April 23, 2023

**1. Definition for sin and cos of an angle**

For any angle  $\alpha$ , we define two numbers: (sine)  $\sin\alpha$  and (cosine)  $\cos\alpha$  as the length of the two legs (catheti) in a right triangle when the hypotenuse of the triangle is 1.

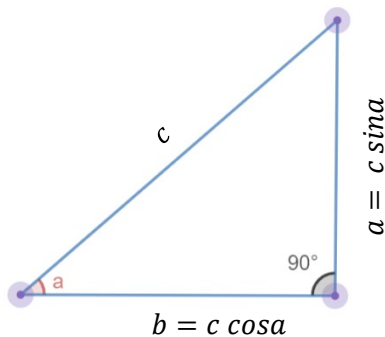


In general, for a right-angle triangle with hypotenuse not equal to 1, the  $\sin\alpha$  and  $\cos\alpha$  of the angle are defined as:

$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

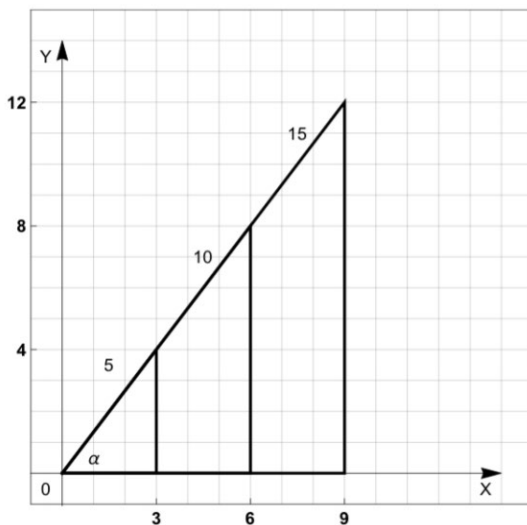
This is because the definitions on  $\sin$  and  $\cos$  do not really depend on size of the triangle, but only the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle  $\alpha$  and hypotenuse  $c$ , then the sides will be  $c \sin\alpha$  and  $c \cos\alpha$ :



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{c \sin\alpha}{c}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{c \cos\alpha}{c}$$

**Example:** Consider the angle  $\alpha$  in the following triangles:



$$\sin\alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$\cos\alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

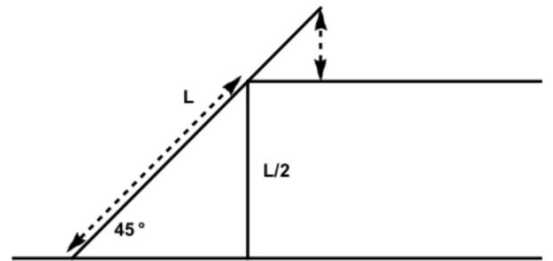
## 2. Table with values for trigonometric functions

Function	Notation	Definition	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
sine	$\sin(a)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(a)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

### Homework problems

All angles are measured in degrees.

- A tree casts a 60 m long shadow when the angle of elevation of the sun is  $30^\circ$ . How tall is the tree? [Angle of elevation is the angle that line from tip of shadow on ground to top of tree makes with the horizontal.]
- A ladder of length  $L$  is resting on a ledge whose height is half of the ladder's length. The ladder makes a  $45^\circ$  angle with the ground. Express answers in terms of  $L$ .
  - How long is the portion of the ladder between the ground and the point of contact of ledge and ladder? [indicated by a long dashed arrow]
  - At what height is the top of ladder above the ledge? [indicated by short dashed arrow - this is another right triangle.]



- A cruise ship travels north for 3 miles and then north-west for another 3 miles. How far will it end up from its original position (from the start to the end point). [Note: North-east is the direction that bisects the angle between north and east.]
- A ship travels for 3 miles north, then turns and goes for 2 miles northeast, then for another 5 miles north-northeast. Where will it be at the end - how far east and north of the original position? [Northeast means that its direction bisects the angle between north and east directions, thus forming an angle of  $45^\circ$  with due north. North-northeast means that this direction bisects the angle between north and north-east, thus forming  $22.5^\circ$  angle with due north.]
- Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? [Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.]
- (\*) Consider a parallelogram ABCD with  $AB = 1$ ,  $AD = 3$ ,  $\angle A = 40^\circ$ . Find the lengths of diagonals in this parallelogram.
- Prove that the area of a triangle ABC can be computed using the formula  $A = \frac{1}{2} \cdot AB \cdot AC \cdot \sin \angle A$ . [Hint: what is the altitude from vertex B?]
- What is the area of a regular pentagon inscribed in a circle of radius 10? [Make sure to use a trigonometric function.]

