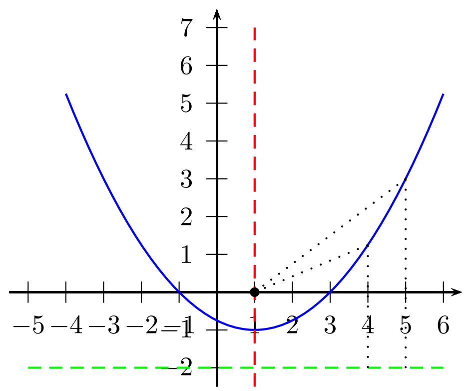
Math 7: Handout 20 [2023/03/12]
Coordinate Geometry 3: Parabolas. Addition of Graphs

Properties of a Parabola

A parabola is the set of all points in a plane that are equally distant away from a given point and a given line (see black dotted lines).

This given point is called the **focus** (black dot) of the parabola and the line is called the **directrix** (green line).

If the parabola is of the form $(x-h)^2 = 4p(y-k)$, the vertex is (h,k), the focus is $(h, k+p)$ and directrix is $y = k-p$



The graph above is a parabola with vertex $(1,-1)$, focus $(1,0)$, and the directrix $y=-2$. It has thus equation $(x-1)^2 = 4(y+1)$ or $y = \frac{1}{4}(x-1)^2 - 1$.

In general, the plot of a quadratic function $y = ax^2+bx+c$ is parabola; to plot it, complete the square and find its vertex, e.g.:

$$y = 2x^2+12x-10 = 2(x^2+6x-5) = 2\big((x+3)^2-14\big) \text{ or } \frac{1}{2}(y+28) = (x+3)^2,. \tag{1}$$

This parabola has vertex $(-3,-28)$ and focal distance $\frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}$.

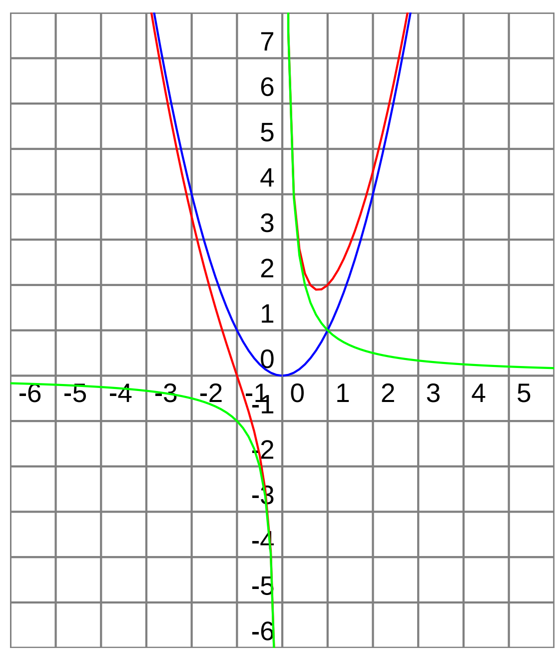
Adding Graphs

Now that we know how to draw a lot of basic graphs and how to use transformations, we can draw more complicated graphs — that is, graphs that are we get by adding two functions.

For example, if we want to draw a graph of a function

$$y = x^2 + \frac{1}{x}$$

we can carefully examine graphs of $y = x^2$ (blue) and $y = 1/x$ (green), and then see what happens if one adds these two graphs (red).



HOMEWORK

1. Sketch the following functions:

(a) $y = |x| + |x+1|$

(b) $y = |x-1| + |x+1|$

(c) $y = |x-1| - |x+1|$

(d) $|y| = x$

[Hint for this problem and the next one: Draw the graphs of each of the summands separately, and then try to add the graphs.]

2. Sketch the following functions:

(a) $y = x + \dfrac{1}{|x|}$

(b) $y = \sqrt{x} + x$

(c) $y = 2x - \dfrac{1}{x}$

3. Graph $x^2 = 4y$. What is the focus, directrix and vertex of the parabola?

***4.** Find all intersection points of parabola $y = x^2$ and the circle with radius $\sqrt{6}$ and center at $(0,4)$.

***5.** Let A and B be points with coordinates (a,r) and (b,s). Then let N be the point with coordinates $(b-a, s-r)$, and let O be the origin $(0,0)$. Show that $ON \cong AB$ and that $ABNO$ is a parallelogram (Hint: the diagonals AN, BO must bisect each other.)