<u>Homework 4: Algebraic operations part2</u>

HW4 is Due October 26; submit it to Google classroom 15 minutes before the class time.

Today we discussed how one works with algebraic expressions in fractional form, i.e. rational expressions containing variables and unknowns, such as $2(x + 1)^2$, in the denominator. I would like you to practice again using the main algebraic identities and exponent laws - it is important to learn using them. Here are the formulas about exponents, radicals, and the basic algebraic identities which you will be using.

1. **Exponents Laws**
   If a is a real number, n is a positive integer $\qquad a^n = a \times a \times \cdots \times a$

   $$\underbrace{\qquad\qquad\qquad}_{\text{n-times}}$$

   $$a^0 = 1$$

   $$a^m \times a^n = a^{m+n}$$

   $$a^m \div a^n = a^{m-n}$$

   $$(ab)^n = a^n b^n$$

   $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

   $$a^{-n} = \frac{1}{a^n}$$

   $$(a^m)^n = a^{mn}$$

2. **Radicals**

   $$a^{\frac{m}{n}} = \sqrt[m]{a^n}$$
   $$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. **Main Algebraic Identities**

   $$(a + b)^2 = a^2 + 2ab + b^2$$

   $$(a - b)^2 = a^2 - 2ab + b^2$$

   $$a^2 - b^2 = (a - b)(a + b)$$

   <u>*Homework problems on the next page*</u>

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer and some justification for why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that anyone can read them and follow your arguments.

1. Simplify

   a. $\sqrt{\dfrac{56}{13}} \cdot \sqrt{\dfrac{26}{7}} =$

   b. $\sqrt{48} =$

   c. $\dfrac{\sqrt{48}}{\sqrt{15}} =$

2. Express the following expressions in the form $2^r 3^s a^m b^n$:
   a. $8a^3 b^2 (27a^3)(2^5 ab) =$
   b. $3^2 (2ab)^3 (16a^2 b^5)(24b^2 a) =$
   c. $16a^2 b^3 (6ab^4)(ab^2)^3 =$

3. Expand as sums of powers of x:
   a. $(2x + 5)^2 =$
   b. $(2 - 4x)^2 =$
   c. $(1 - 2x)^2 =$

4. Factor (i.e., write as a product) the following expressions:
   a. $4x^2 + 8xy + 4y^2$
   b. $9x^2 - 25$
   c. $(x - 2)^2 - (y + 3)^2$
   d. $256 - a^8 b^8$
   e. $(x - 2)^2 - 10(x - 2) + 25$
   f. $3x^3 - x^2 y + 6x^2 y - 2xy^2 + 3xy^2 - y^3$
   g. $a^2 - b^2 - 10b - 25$
   h. $x^4 + 4$ Hint: add and subtract $4x^2$

5. Solve the following equations. :
   a. $5(x + 1) = 3x + 2$
   b. $(x^2 - 1)(x + 2) = 0$
   c. $\dfrac{x+2}{x+3} = 2$
   d. $(x - 3)(x + 4) = 0$
   e. $x^2 + 4x = 0$
   f. $x^3 + 4x = 0$

6. Prove:
   a. $(a + b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$
   b. $(a - b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3$
   c. $Find (a + b)^4, (a - b)^4$ using the previous results

7. A $4 \times 4 \times 4$ cubical box has 64 small cubes inside. How many of these touch a side or the bottom of the box?

8. Amanda has an average of 92 on her seven tests. What should she get on her $8th$ test to have an average of 93?