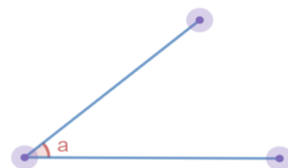
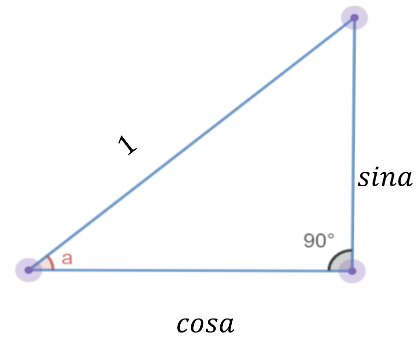
Homework 25: Trigonometry, basic definitions.

HW25 is Due April 2; submit it to Google Classroom 15 minutes before the class time.

1. Definition for sin and cos of an angle

For any angle α, we define two numbers: (sine) $sin\alpha$ and (cosine) $cos\alpha$ as the length of the two legs (catheti) in a right triangle when the hypothenuse of the triangle is 1.

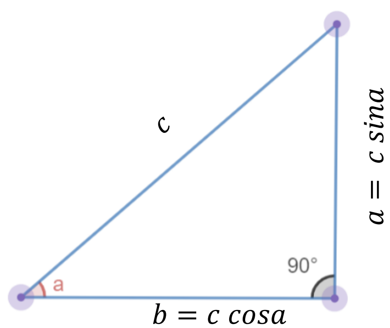




In general, for a right-angle triangle with a hypothenuse not equal to 1, the ***sina*** and ***cosa*** of the angle are defined as:

$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} \qquad cosa = \frac{\text{adjacent side}}{\text{hypothenuse}}$$

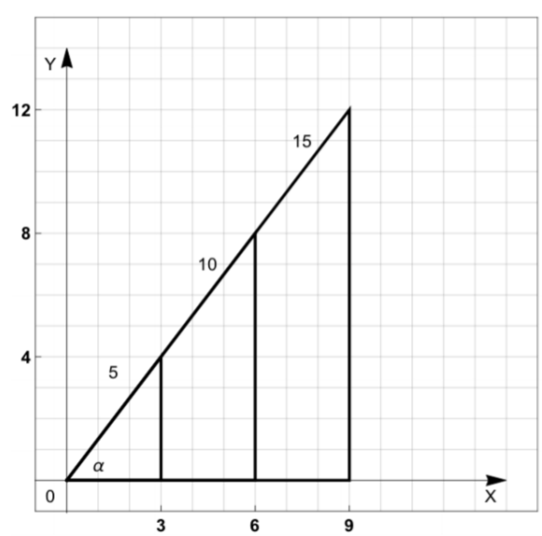
This is because the definitions of ***sin*** and ***cos*** do not depend on the size of the triangle, but only on the angle itself. Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle **α** and hypotenuse **c**, then the sides will be $c \sin\alpha$ and $c \cos\alpha$:



$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{c\ sina}{c}$$

$$cosa = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{c\ cosa}{c}$$

Example: Consider the angle α in the following triangles:



$$sina = \frac{\text{opposite side}}{\text{hypothenuse}} = \frac{4}{5} = \frac{8}{10} = \frac{12}{15}$$

$$cosa = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{3}{5} = \frac{6}{10} = \frac{9}{15}$$

2. Table with values for trigonometric functions

Function	Notation	Definition	0^0	30^0	45^0	60^0	90^0
sine	sin(a)	$\frac{\text{opposite side}}{\text{hypothenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	cos(a)	$\frac{\text{adjacent side}}{\text{hypothenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Homework problems

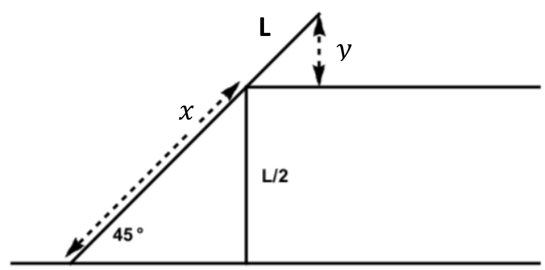
Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

All angles are measured in degrees.

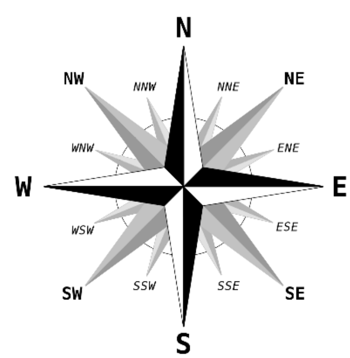
1. Which one is greater?
    a. 0 or sin 0^0
    b. 1 or sin 30^0
    c. sin 45^0 or cos 45^0
    d. cos 60^0 or sin 30^0

2. A tree casts a 60 m long shadow when the angle of elevation of the sun is 30^0. How tall is the tree? (Angle of elevation is the angle that a line drawn from the tip of the shadow on the ground to the top of the tree makes with the horizontal.)

3. LA ladder of length **L** is resting on a ledge whose height is half of the ladder's length. The ladder (the solid line) makes a 45^0 angle with the ground. Express answers in terms of L.
    a. How long is the portion of the ladder between the ground and the point of contact of the ladder with the ledge? (x indicated by a long dashed arrow)
    b. At what height is the top of the ladder above the ledge? (y indicated by short dashed arrow - this is another right triangle.)



4. A cruise ship travels north for 3 miles and then northwest for another 3 miles. How far will it end up from its original position (from the start to the end point)? (Note: Northeast is the direction that bisects the angle between north and east.)



5. A ship travels for 3 miles north, then turns and goes for 2 miles northeast, then for another 5 miles north-northeast. Where will it be at the end - how far east and north of the original position? (Northeast means that its direction bisects the angle between north and east directions, thus forming an angle of 45^0 with due north. North-northeast means that this direction bisects the angle between north and northeast, thus forming a 22.5^0 angle with due north.)

More on the next page →

6. Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.

7. (*) Consider a parallelogram ABCD with AB = 1, AD = 3, $\angle A = 40^0$. Find the lengths of diagonals in this parallelogram.

8. (*) Prove that the area of a triangle ABC can be computed using the formula $A = \frac{1}{2} \cdot AB \cdot AC \cdot \sin \angle A$. Hint: what is the altitude from vertex B?

9. What is the area of a regular pentagon inscribed in a circle of radius 10? Make sure to use a trigonometric function in your calculations.