MATH 8: HOMEWORK 02
REVIEW II

All starred problems are optional. If you have time, do them for fun.

1. Consider the following quadratic equation:

$$x^2 + 13x + 30 = 0$$

(a) What is the discriminant of this equation?
(b) Sketch a graph of this quadratic polynomial using completing the square method.
(c) Solve the equation.

2. Let $x + y = 10$ and $xy = 15$
(a) Calculate $x^2 + y^2$.
(b) Calculate $(x - y)^2$.
(c) Calculate $\frac{1}{x} + \frac{1}{y}$.

3. Let x_1 and x_2 be the solutions of $x^2 - 12x + 19 = 0$. Without solving the equation find the value of the expression below(Hint: write the expression in terms of $x_1 + x_2$ and x_1x_2)

$$x_1(1 - x_1) + x_2(1 - x_2).$$

4. Write down the following fraction in a form $a + b\sqrt{5}$:
(a) $\frac{1}{\sqrt{3}-2}$
(b) $\frac{7}{1+\sqrt{3}}$
(c) $\frac{9-3\sqrt{5}}{\sqrt{5}-2}$

5. Solve the equation:

$$|3x - 8| = 10$$

6. Solve the following inequality. Write your answer as a set of possible values for x.

$$\frac{(x + 2)^2(x - 7)}{x + 3} \leq 0$$

7. Find the largest number among them $\sin 30° \times \cos 30°$, $\sin 45° \times \cos 45°$, $\sin 60° \times \cos 60°$.

8. If a right triangle ΔABC has sides $AB = 3\sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.

9. A florist has 20 fresh roses. A child wants to buy a bouquet of three. How many options has the florist to create a bouquet ?

10. There are 10 girls and 16 boys in the class. How many different five-member teams could be formed if each team should be composed of two girls and three boys ?

***11.** A flag is made of three horizontal strips of fabric with solid colors, either red, white, or blue. If no two adjacent strips can be the same color, how many distinct flags can be sewn?(Hint: Start counting from the middle)

***12.** A teenager placed three letters randomly into three envelopes. What is the probability that at least one of the recipients gets his letter ? (Hint:Count both total and favorable cases)

***13.** (Optional Geometry Review: Law of Cosines) Consider a parallelogram $ABCD$ with $AB = 1$, $AD = 3$, $\angle A = 40°$. Find the lengths of diagonals in this parallelogram.

***14.** (Optional Vectors Review)A cruise ship travels North for 3 miles and then North-West for more 3 miles. How far will it end up from its original position? [North-West is the direction that bisects the angle between N and W.]

$$\text{Recall that } (u \pm v)^2 = u^2 \pm 2uv + v^2, \text{ and } z^2 - y^2 = (z-y)(z+y)$$

Of course that not all quadratic equations are perfect squares in rational numbers, but

The **Determinant formula** can be obtained by **"Completing the Square"**

$$ax^2 + bx + c = 0$$

If $a = 1$, we use a trick $2\frac{b}{2} = b$ to create a perfect square

$$\begin{aligned} x^2 + bx + c &= x^2 + 2\tfrac{b}{2}x + c = \left(x^2 + 2\tfrac{b}{2}x + \tfrac{b^2}{2^2}\right) - \tfrac{b^2}{2^2} + c \\ &= \left(x + \tfrac{b}{2}\right)^2 - \tfrac{b^2-4c}{4} = \left(x + \tfrac{b}{2}\right)^2 - \tfrac{D}{4} \end{aligned} \tag{1}$$

where $D = b^2 - 4c$.

Determinant formula when a is not equal to 1.
We have $ax^2 + bx + c = 0$ equivalent to

$$\left(x + \tfrac{b}{2a}\right)^2 = \tfrac{D}{4a^2}, \qquad D = b^2 - 4ac$$

Therefore, if $D < 0$, there are no solutions in real numbers; if $D \geq 0$, solutions are

$$x + \tfrac{b}{2a} = \pm\frac{\sqrt{D}}{2a}, \text{ or equivalently } x = \frac{-b \pm \sqrt{D}}{2a} \tag{2}$$

Example:

$$x^2 + 3x + 1 = x^2 + 2 \cdot \tfrac{3}{2}x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 1 = x^2 + 2 \cdot \tfrac{3}{2}x + \tfrac{9}{4} - \tfrac{3}{4} = (x + \tfrac{3}{2})^2 - \tfrac{3}{4}$$

This is a difference of two squares $u^2 - v^2$, with $u = (x + \frac{3}{2})$ and $v = \frac{\sqrt{3}}{2}$. Then we can rewrite:

$$x^2 + 3x + 2 = (x + \tfrac{3}{2} + \tfrac{\sqrt{3}}{2})(x + \tfrac{3}{2} - \tfrac{\sqrt{3}}{2})$$

thus, $x^2 + 6x + 2 = 0$ iff $x + 3 + \sqrt{7} = 0$, which gives $x = -3 - \sqrt{7}$, or $x + 3 - \sqrt{7} = 0$, which gives $x = -3 + \sqrt{7}$.

Vieta Formulas allow us to find the sum and the product of roots of a quadratic equation without explicitly calculating them.

$$x_1 + x_2 = -\frac{b}{a}, \text{ and } x_1x_2 = \frac{c}{a}$$

If $a = 1$, we have: $x_1 + x_2 = -b$ and $x_1x_2 = c$
Why? If x_1, x_2 are the two roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$ax^2 + bx + c = a(x - x_1)(x - x_2).$$

By rewriting we get

$$a(x - x_1)(x - x_2) = ax^2 - a(x_1 + x_2)x + ax_1 \cdot x_2$$

Two quadratics are equal if their coefficients are, so we get

$$b = -a(x_1 + x_2), \text{ and } c = ax_1 \cdot x_2$$